Kkk. 467

THE KEY OF THE MATHEMATICKS

25.31

New Forged and Filed:

Together with

A Treatise of the Resolution of all kinde of Affected Æquations in Numbers.

With the Rule of Compound Usury; And demonstration of the Rule of false Position.

And a most easie Art of delineating all manner of Plaine Sun-Dyalls. Geometrically taught

BY

WILL. OUGHTRED.

LONDON,
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1807: 17

TO THE HONORABLE,
SIR *RICHARD ONSLOW*
KNIGHT, ONE OF THE
KNIGHTS OF THE PARLI-
AMENT FOR THE COUNTY
OF *SURREY.*

AND TO *ARTHVR ONSLOW*
ESQUIRE, ONE OF THE
BURGESSES OF THE
PARLIAMENT:
ELDEST SON OF THE SAID
SIR *RICHARD ONSLOW.*

WILLIAM OVGHTRED RECTOR
OF *ALBVRY* IN THE SAID
COUNTY OF *SURREY,*
IN TESTIMONY OF THE HO-
NOUR AND RESPECTIVE OBSER-
VANCE HE BEARETH TO THAT
NOBLE FAMILY, PUBLISH-
ETH AND DEDICATETH
THESE HIS ENSUING
TREATISES.

To the Reader.

MAny yeares ſince, I being imployed by the late illuſtrious Earle of Arundel, *to inſtruct one of his ſons in the Mathematicks, penned for his uſe in Latine, a method of precepts for the more ready attaining, not a ſuperficial notion, but a well-grounded underſtanding of thoſe myſterious Sciences, and of the ancient Writers thereof. This afterward at the requeſt of divers learned and judicious men, I publiſhed under the name of* Clavis Mathematicæ. *Which Treatiſe being not written in the uſuall ſyntheticall manner, nor with verbous expreſsions, but in the inventive*

way of Analitice, *and with ſymboles or notes of things inſtead of words, ſeemed unto many very hard; though indeed it was but their owne diffidence, being ſcared by the newneſſe of the delivery; and not any difficulty in the thing it ſelfe. For this ſpecious and ſymbolicall manner, neither racketh the memory with multiplicity of words, nor chargeth the phantaſie with comparing and laying things together; but plainly preſenteth to the eye the whole courſe and proceſſe of every operation and argumentation.*

Now my ſcope and intent in the firſt Edition of that my Key was, and in this New Filing, or rather forging of it, is, to reach out to the ingenious lovers of theſe Sciences, as it were Ariadnes *thread, to guide them through the intricate Labyrinth of theſe ſtudies, and to direct them for the*

more

more eaſe and full underſtanding of the beſt and antienteſt Authors; ſuch as are Euclides, Archimedes, Apollonius Pergæus *that Great Geometer*, Diophantus, Ptolomæus, *and the reſt: That they may not only learn their propoſitions, which is the higheſt point of Art that moſt Students aime at; but alſo may perceive with what ſolertiouſneſſe, by what engines of Æquations, Interpretations, Comparations, Reductions, and Diſquiſitions, thoſe antient Worthies have beautified, enlarged, and firſt found out this moſt excellent Science.*

Truly when I was converſant in reading their bookes, and with wonder obſerved their moſt witty demonſtrations, ſo skilfully framed out of principles, as one would little expect or thinke, but laid together with divine Artifice: I was even amazed, whence poſſibly any power of imagination

ſhould

ſhould be able to ſuſtaine ſo immenſe a pile of conſequences, and cauſe that ſo many things, ſo far aſunder diſtant, could be at once preſent to the minde, and as with one conſent joyne and lay themſelves together for the ſtructure of one argument. Wherefore that I might more cleerly behold the things themſelves, I uncaſing the Propoſitions and Demonſtrations out of their covert of words, deſigned them in notes and ſpecies appearing to the very eye. After that by comparing the divers affections of Theoremes in equality, proportion, affinity, and dependence, I tryed to educe new out of them. Laſtly, by framing like queſtions problematically, and in way of Analyſis, as if they were already done, reſolving them into their principles, I ſought out reaſons and means whereby they might be effected. And by this courſe of practice, not without

long

long time, and much induſtry, I found out this way for the helpe and facilitation of Art.

Hereunto alſo is added a ſhort Tract concerning the Reſolution of adfected Æquations propoſed in numbers, whereof mention is made in the end of the former Booke. *And by the way is inſerted, ſo much as is needfull, for the uſe both of Decimall parts, and of Logarithmes. And laſt of all, a moſt naturall and eaſie Geometricall way of delineating Sun-Dyals, upon any plaine howſoever ſituated.*

I *was unwillingly drawne, at this my declined age, to appear unto the world in ſuch a kinde of Subject.* But *occaſion was adminiſtred by one Mr.* Seth Ward, *a young man excellently accompliſhed with all parts of polite Literature, then Fellow of* Sidney *Colledge in* Cambridge, *who tooke the*

pains

pains to seek me out at my house, and by a gentle violence induced me to publish again my former Tractate in a manner new moulded and perfected: And also divers pieces (among many) which I had long agoe commented and digested, some compleatly, and some more rudely, without any intent to make the same publick.

So that whatsoever fruit any one shall reape hereby, it is to bee acknowledged to the said Mr. Ward, *as the first motive, and next unto my very loving and learned friends Mr.* William Leech, *Mr.* William Brearly, *and Mr.* Robert Wood, *who have been ready and helpfull incouragers of me in this labour: And lastly (but indeed chiefly) unto my knowing and worthy friends Mr.* Ionas Moore, *who is singularly learned, painfull, and curious, as well in this way, as in other the Mathema-*

tical

tical Sciences, and Mr. Thomas VVharton, who (as he is no meane proficient in those studies, so he) may with successe serve this age in the necessary mysteries of Botanicke, Anatomick, and Hermetick learning, by him medically imployed, who have not only examined the operations in these Treatises, and amended such escapes, as by reason of mine old age, multiplicity of businesse, and distraction of my mind, through unspeakable wrongs done to me (by whom I have least deserved any) might happen: but also have (even by the Printers testimony) bestowed exceeding great paines and expense, in correcting the Presse: which (indeed) was my chiefest care; and without which, Bookes of this kind are extreamly troublesome, if not useless to the Reader.

The Printer desires the Courteous Reader to amend these faults escaped in the sheet E, *by a mistake in printing it off, before it was fully corrected.*

Page 50. line 20. for √c343.9. read √c343.7. p. 58. & 59. where you finde a Coma fixed to the end of Z. r. Z. p. 59. l. 10. for Zq. r. Zq. p. 62. l. 11. for Aqhq. r. Hqhq. p. 63. l. 3. for P r. A. p. 123. l. 37. for 25. r. 15. p. 124. l. 9. blot out, *the side of*.

THE KEY OF THE MATHEMATICS NEW FILED.

CHAP. I.

OF NOTATION.

1 A Table very uſefull, not onely for Notation of numbers which at the very firſt view, it ſheweth: but alſo in all manner of computation by numbers, whether common or figurate, or artificiall, which are uſually called Logarithmes.

Integers						Unit Partes.								
9	8	7	6	5	4	3	2	1	0	$\frac{-}{1}$	$\frac{-}{2}$	$\frac{-}{3}$	$\frac{-}{4}$	$\frac{-}{5}$	$\frac{-}{6}$	$\frac{-}{7}$	$\frac{-}{8}$	$\frac{-}{9}$
M	MMM		MMM				C	X	I	X	C	MMMMMM						
M	MMM		C	X	I								XCMMMM					
M	C	X	I											XCM				

2 In this Table the upper numbers are Indices or Exponents of termes continually proportionall from an unit both wayes: and are in Integers, or whole numbers, adfirmative; in parts or fractions negative.

The Progreſſion is in decuple *ratio* or proportion toward the left hand; and in ſubdecuple toward the right hand: according as the numerall letters doe ſhew. The Progreſſion therefore from the unit in Integers is 1, 10, 100, 1000, 10000, &c. and in parts, $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10000}$, &c.

3 And after this manner in all other Progreſſions, whether the termes increaſe, or decreaſe, their proper Indices are to be ſet over them.

4 I have indeed made choice to frame the table in decuple proportion, both that the valors of all numbers (be they Integers, Parts, or mixed) may be eſtimated by grades and periods; and alſo becauſe this decimall Logiſtica or accounting is for Aſtronomicall calculations much eaſier and neater than that Sexagenarie commonly in uſe; which he plainly perceived, whatſoever he was, that firſt reduced the Canon of Sines from a Semidiameter of 60 parts unto 1, with circles or ciphers annexed. I wiſh the ſame alſo were done in all other Aſtronomicall Canons.

5 Decimall parts are written in one line, with the Integers; and are diſtinguiſhed with a ſmall rectangular line which therefore is called *Separatrix*, or the ſeparating line. And as in Integers every degree, from the unite place is increaſed toward the left-hand by decupling: ſo in decimall parts every degree is diminiſhed toward the right hand by ſubdecupling.

6. Decimall parts take their denomination from the place of their laſt figure: as 0,5 are 5 tenth parts

parts : 0,56 are 56 hundred parts, 0,056 are 56 thouſand parts : and ſo if there were more places of parts.

7. Circles before Integers, and after decimall parts, are of no value : but after Integers, and before decimall parts (that is, being on either ſide next to the ſeparating line) they retaine their owne force: for they conſtitute degrees, by which the valors of the figures are eſtimated : as 0005 ſignifie only 5: and 0,5000 ſignifie 5 tenth parts.

8. Wherefore in writing decimall parts the ſeparating line is evermore to be ſet down: and if any places happen to be void, they are to be filled up with circles : 0,00005 are 5 hundred thouſand parts.

9. The ſigne of adding or of adfirmation is + plus, or pl: as 34 or +34.

10. The ſigne of diminiſhing or of Negation, is — minus, or mi: as − 34, are denied to be at all.

11. The ſigne ſtill belongeth to the magnitude following, before which it is ſet. And every magnitude, before which the ſigne of negation is not prefixed, is underſtood to be adfirmative, and to have the ſigne +, though it be not expreſſed.

12. And note, that I uſe the ſignes + and − when one ſingle magnitude is adfirmed or denied of another ſingle one; but I uſe the ſignes of pl, and mi, when a compound magnitude is adfirmed or denied of one ſingle : or contrariwiſe a ſingle of a compound.

13. Magnitudes may be ſignified, either by numbers expreſſing their meaſure: or alſo by Species. as a line 7 inches long, may bee ſet downe either by the figure 7: or by ſome one letter or note, A,B,C,&c. or by two letters placed at the ends thereof, AB, BC, CD, &c; at pleaſure: ſo that you keep in mind for what magnitude every Species is ſet.

14. This *Specious Arithmetic* is more appliable to the Analyticall art, (in which by taking the thing ſought as knowne, we finde out that we ſeeke) than that Numerous. For in the Numerous, the numbers with which we worke, are ſo, as it were, ſwallowed up into that new which is brought forth, that they quite vaniſh, not leaving any print or footſtep of themſelves behinde them. But in the Specious, the ſpecies remaine without any change, ſhewing the proceſſe of the whole worke: and ſo doe not onely reſolve the queſtion in hand; but alſo teach a generall Theoreme for the ſolution of like queſtions in other magnitudes given.

CHAP. II.

OF ADDITION.

1. THE number found by Addition, is called the Summe, or the Aggregate. as 3 and 7 are 10.

2. Addition begins at the right hand, and writeth the

the particular Summes of the severall rowes underneath every one in its owne place.

3. In Addition, all the numbers given being put together, are equall to the Summe.

Examples of Addition.

79403	3794|1236	17 l.	13 s.	4 d.
8956	5841	3	9	16	7
67293	9471	08	238	09	6
5087	4720	17439	70	00	10
160739	48	15	40	10	3
	100941	8599	384	10	6

4. *Specious* Addition joyneth together all the magnitudes given, keeping their signes.

to 3A	A	5A	3A	A
adde A	-A	-3A	-5A	E
Summe 3A+A	A-A	5A-3A	3A-5A	A+E
that is 4A	0	2A	-2A	

to A+B	A+B	
adde A-B	A-C	So in the Addition of Indices.
Summe 2A	2A+B-C	

3	3
2	2
1	1

CHAP. III.
OF SUBDUCTION.

1. THE number found by Subduction, is called the Remaines, the Reſt, the Difference, or the Exceſſe: as out of 7 take 3, there remaines 4.

2. Subduction begins at the right hand, and writeth the particular differences of the ſeverall rowes, underneath every one in his owne place.

3. In Subduction, the number to bee ſubducted, together with the difference, is equall to the number out of which.

Examples of Subduction.

347206836	37941236	171.	13 s.	4 d.
6807592	947108	9	16	7
340399244	28471156	7	16	9

4. *Specious* Subduction joyneth together both the magnitudes, changing all the ſignes of the magnitude to be ſubducted.

out of	4A	3A	5A	A
take	A	5A	-3A	E
Remains	4A-A	3A-5A	5A+3A	A-E
that is	3A	-2A	8A	
out of	A	A	So in the	3 | 3
take	B+C	B-C	Subduction	2 | 2
Remains	A-B-C	A-B+C	of Indices.	5 | 5

CHAP.

CHAP. IV.
OF MULTIPLICATION.

1. THE number found out by Multiplication, is called the Factus, the Product, the Rectangle, or the Plaine. For one of the numbers propoſed is taken for the Longitude; the other for the Latitude. And the numbers ſo multiplied are called the Factores and the Sides. For the greateſt power of two magnitudes is a quadrangular figure contained under them, having all right angles, and the oppoſite ſides parallel.

2. Multiplication begins at the right hand, and draweth (that is, multiplieth) every figure of one number given, into every figure of the other number ſeverally: and at the laſt gathereth together the ſeverall products into one ſumme, having diligent care of the due places. And if decimall parts bee mixed with the numbers propoſed, ſo many of the laſt figures are to be cut off with a ſeparating line, as there were places of parts in both the Factores. For in Multiplication the Index of every particular figure of the Product is found by adding together the Indices of the figure multiplying and the figure multiplied. Thus 58,73 being drawne into 600, maketh 35238: for the Index of the figure 6 in 600, is 2: and the Index of the laſt figure 3 in 58,73, is $\bar{2}$. Adde the

the two Indices 2 and $\bar{2}$; there will be 0 for the Index of 8 the last figure of the product 35238: which therefore pertaineth to the place of units. And the like manner of valuation may be made of every particular figure of the product.

3. If, of the numbers proposed, one or both have circles joyned to them on the right hand: leave out the circles and so multiply: lastly, take into the product so many places more of Integers, as there were circles left out in both the *Factores*.

4. In Multiplication, As an unit is to one of the numbers multiplied, so is the other to the product: as if 4 be drawn into 6, the product will be 24: wherfore 1. 4::6.24. or 1.6::4.24.

Examples of Multiplication.

4576	58034	358
892	475	600
9152	290170	224800
41184	406238	
36608	232136	5873
4081792	27566150	600
		35238

5. A Contraction of Multiplication very usefull in Logistica, is thus; If it will serve your purpose to have the product not intire, but diminished by cutting off some of the last figures: for the unit place of the

the lesser number under that figure of the greater, whose Index is equall to the number of figures, either to be cut off in the Integers; or to be left in the decimall parts: and the other figures of the lesser number set under the greater number, but the quite contrary way. Then in multiplying, beginne still at the figure of the greater number, which stands above that figure of the lesser number wherewith you multiply: yet having a regard to that increase which would arise out of the following figures of the greater number. Of this abbreviation there be foure Cases.

Case I. If you would have the product pure from all parts: set the unit place of the lesser, under the unit place of the greater. As in the example, 246,914 being multiplied into 35,27, produceth 8708 Integers, without any decimall parts at all.

```
246,914
72,53
-------
 7407
 1235
   49
   17
-------
 8708
```

Case II. If you would have the product, with certaine places of parts, suppose foure: set the unit place of the lesser number, under the fourth place of parts in the greater. As in the former example the product shall bee 8708,6568, mixt with foure places of parts.

```
246,914
     72,53
---------
74074200
12345700
  493828
  172840
---------
8708,6568
```

Case III.

Case III. If you would have the product shortened by certain places of Integers, suppose five: Set the unit place of the lesser number, under the fifth place, before the unit place of the greater. As in the example, 80902, the sine of Grad: 54, is to be multiplied by 39875 the sine of Gra: 23 30 (which is the Sunnes greatest declination) the product will bee 32266, the sine of the Sunnes declination in ♉ 24°.

80902	
57893	
24271	
7281	
647	
63	
4	
32266	

Case IV. If you would have the product shortened by certaine places of Integers, suppose five, to be repaired againe with some places of parts, suppose foure. Because $5 + \bar{4} = 1$: Set the unit place of the lesser number one place, before the unit place of the greater: as in the example, the sine 42262 is multiplied into $0,\underline{0}0064$, so that the five last figures of the product being cut off, there may bee restored foure places of parts. The product shall be $0,\underline{0}0027$.

42262	
46000	
25	
2	
$0,\underline{0}0027$	

6. Specious Multiplication joyneth together both the magnitudes proposed, with the note, into, or in, or ×, or for the most part without any note at all, if the magnitudes be set downe only with one letter. And if the signes of both be alike, the product shall be

be adfirmative; but if they be diverſe, it ſhall bee negative. And it is declared by this word, into. And note that A in A, or A×A, or A·A is Aq. AAA, or AqA, is Ac. AAAA, or AqAq, or AcA, is Aqq. AAAAA, or AcAq, or AqqA, is Aqc. AAAAAA, or AcAc, or AqqAq, or AqcA, is Acc. &c For every upper *poteſtas* (or power) is made of two inferiour, whoſe dimenſions put together are equall to the dimenſions of that upper. As many as the magnitudes multiplied are, ſo many are the dimenſions of their product.

Draw A	A+E	A−E	A+E+I	B+I
into E	B	B	Z	A
Product AE	BA+BE	BA−BE	ZA+ZE+ZI	BA+A

Draw 3A	AE	AE	A+E	A+E
into 2A	A	AE	A+E	A−E
Product 6Aq	AqE	AqEq	Aq+AE	Aq+AE
			+AE+Eq	−AE−Eq
			Aq+2AE+Eq	Aq — Eq

After this manner, magnitudes conſiſting of two letters, are multiplied. As if the ſide AB+CD be to be drawne into it ſelfe; the Product ſhall be ABq+2AB·CD+CDq: which is the quadrate thereof.

CHAP.

CHAP. V.
OF DIVISION.

1. THE number found out by Diviſion is called the Quotient, or alſo Parabola, that is the Application: becauſe it ariſeth out of the application of a Plaine to a longitude given, to finde out a Latitude agreeing thereto. And note, that if a number be applied to another number, with a ſmall line drawne between them, it ſheweth that the upper number is to be divided by the under number unto which it is applied: as $\frac{12}{4}$, and $\frac{15}{3}$.

2. Diviſion begins at the left hand, and having out of the formoſt figures of the whole dividend diſtinguiſhed for the firſt dividuall, a number not leſſe than the diviſor, and written the diviſor under it, or at leaſt imagined it to be underwritten; taketh every ſeverall figure of the diviſor, out of the figures ſtanding above, equally, as often as may be. Then having multiplied the diviſor into the quotient found, and ſubducted the product out of the dividuall, it ſetteth forward the diviſor into the next place: and in the very ſame manner frameth a new diviſion: and ſo proceeds till it hath gone over the whole dividend. Now every particular Quotient found, muſt be of the ſame place or degree, of which the figure of the dividend is, which ſtandeth, or is imagined to ſtand over the unit place of the diviſor. For in Diviſion

ſion the Index of every particular figure of the Quotient is found, by taking the Index of the figure dividing, out of the Index of the figure divided. So 1 7 1, 4 being divided by 8 5 7+, giveth 0,2 for the Quotient. For the Index of the firſt dividuall figure 1 7, is 1; and the Index of the firſt dividing figure 8, is 2: Take 2 out of 1, there remaineth $\bar{1}$ for the Index of the firſt figure; which therefore pertaineth to the firſt place of the decimall parts.

3. And if the diviſor have circles joyned to him at the right hand, omitting the circles, and cutting off as many of the laſt figures of the dividend, let the diviſion be made in the remaining numbers. In the end of the diviſion, both the circles omited, and the figures cut off, are to be reſtored.

4. In Diviſion, as the Diviſor is to an unit, ſo is the Dividend to the Quotient: or, as the Dividend is to the Diviſor, ſo is the Quotient to an unit. As 24 divided by 6: the Quotient ſhall be 4. Therefore 6.1::24.4. And 24.6::4.1.

5. If a magnitude be produced of two magnitudes, the one of them ſhall meaſure (or divide) it by the other.

6. In Multiplication and Diviſion, an unit alters nothing.

7. If a number multiply a number, and the ſame divide the product, nothing comes of it. For what Multiplication makes up, that Diviſion undoes.

Where-

Wherefore in the application of one magnitude to another, if the same magnitude be both over and under the line, expunge it, in both places.

Examples of Division.

297) 187135075 (630084 127/297

297

1782

893

297

891

2507

297

2376

1315

297

1188

127

1 2

8 9 2 1 3 1 7

297) 1 8 7 1 3 5 0 7 5 (630084 127/297

1 7 8 2 1 3 7 6 8

8 9 2 1 1 8

2 9 0 1

4 3 5 2 5 7

580,34) 2 7 5 6 6, 1 5 0 (47 15

2 3 2 1 1 7 6 8 0

4 0 6 2 3 7

2 9 0 1

6,000) 4320,765

720,1275

8. Some-

8. Sometimes a number is required to be divided by an irrationall or infinit number, whether it be Integer or mixt: And in this case, taking as many of the formost figures of the divisor, as will serve your purpose, for the first divisor, by them divide the number proposed: then for every particular following division, lessen the divisor by cutting of, towards the left hand, so many of the last figures, untill you have found a Quotient large enough. As if 467023 be divided by an infinit number 357,0926425-, the Quotient shall be 1307,85 almost,

```
                    17
                   303
                  2803
                 109930
357,0926425-) 467023 (1307,85-
                357093
                107127
                  2500
                   286
```

This is an excellent contraction of division, and of exceeding great use in Astronomicall computations. As if by 137638, you be to divide 126223 drawne into the whole sine, that is, increased with five circles: put to it, only one circle, and for the four other lessen your divisor; as

137638) 1262230 (91727.

9. *Specious*

9. Spetious Diviſion ſets the dividing magnitude under the dividend, with a line drawne between them; Then conſiders, whether in their compoſition any one magnitude multiplieth both of them, and ſtrikes it out in both. Now Diviſion in the ſame ſignes giveth +, but in diverſe ſignes −. And it is declared by this word, to.

Adplie	AE	BAc	BA+A	BA−CA	6Aq	2×3Aq
to	A	Aq	A	B−C	3A	3A
Quotient	E	BA	B+1	A	2A	2A

ſcil

Chap. VI.

OF PROPORTION.

1. IF of foure numbers given, the firſt bee to the ſecond, as the third is to the fourth: thoſe four are called proportionall numbers. Now the being or habitude of one number to another is found by dividing the Antecedent by the Conſequent, as the Ratio of 31 to 7, is $4\frac{3}{7}$, that is, quadruple ſupertriparting ſevenths.

2. Wherefore, if a number multiply two numbers, the products ſhall be proportionall to the numbers multiplied: And if a number divide two numbers, the Quotients ſhall bee proportionall to the numbers divided. As

4. $\begin{cases} 7.28. \\ 9.36. \end{cases}$ &4) $\begin{matrix} 28(7. \\ 36(9. \end{matrix}$ Alſo A. $\begin{cases} B.\ BA. \\ C.\ CA. \end{cases}$ &A) $\begin{matrix} AB(B. \\ CA(C. \end{matrix}$

3 Where-

Wherefore if foure numbers be propottionall, the product of the two extreames is equall to the product of the two meanes. 7.9::7×4.9×4::28.36. but 7×9×4=9×7×4.

4. From hence followes the Golden Rule (so called) of Proportion. If of three numbers given, the rectangle contained under the second and third, be applied to the first: that is, If the second multiply the third, and the first divide the product: the quotient shall be the fourth proportionall to the three numbers given. Let the three numbers given be 7,9,28: and for the fourth number sought, put Q: therefore 7.9::28.Q. wherefore 7Q=9×28. And therefore $\frac{9\times28}{7}=Q$. Also 5.12::8. $\frac{8\times12}{5}$ that is $19\frac{1}{5}$.

5. Three numbers being given, to finde out the fourth proportionall, the two first doe imply a Ration, and the remainder enters the question, and in direct Proportion the first terme (or Divisor) is *homogene* to that terme by which the question is made: But in Reciprocall Proportion, the first terme (or Divisor) is it selfe that, by which the question is made.

6. Direct Proportion is, when that terme, by which the Question is made, the greater it is, the greater it requires the fourth terme to be; and the lesse it is, the lesser.

7. Reciprocall Proportion is, when that terme by which the Question is made, the greater it is, the lesse

it requires the fourth terme to be: the lesser it is, the greater.

8. Continued proportion ∴, is when all the middle terms between the first and the last, are both consequents and antecedents of Rations: as 8,12,18,27, are ∴ For 8.12::12.18::18.27.

Also $\alpha, \beta, \frac{\beta q}{\alpha}, \frac{\beta c}{\alpha q}, \frac{\beta qq}{\alpha c}, \frac{\beta qc}{\alpha qq}$ &c. are ∴

Wherefore if in this *series*, the last terme be ω, and the summe of all the terms of the whole progression be Z: then $Z-\omega$ will be the summe of all the antecedents; and $Z-\alpha$, the sum of all the consequents.

9. If foure magnitudes be proportionall, $A.\alpha::B.\beta$. they shall also be proportionall being Alterned, and Inverted, and Compounded, and Divided, and Converted, and in Mixture.

$A.\alpha::B.\beta.$

Altern. $A.B::\alpha.\beta.$
Invert. $\alpha.A::\beta.B.$
Compound. $A+\alpha.\alpha::B+\beta.\beta.$
or. $A+B.B::\alpha+\beta.\beta.$
Divid. $A-\alpha.\alpha::B-\beta.\beta.$
or. $A-B.B::\alpha-\beta.\beta.$
Convert. $A.A\pm\alpha::B.B\pm\beta.$
or. $A.A+B::\alpha.\alpha+\beta.$
In Mixture. $A+\alpha.A-\alpha::B+\beta.B-\beta.$
or. $A+B.A-B::\alpha+\beta.\alpha-\beta.$

10. If never so many magnitudes be proportionall, it shall be as one of the antecedents is to its consequent;

quent; ſo the ſumme of the antecedents to the ſumme of the conſequents. If it be $A.\alpha::B,\beta::C.\gamma::D,\delta$: Then will $A.\alpha::A+B+C+D.\alpha+\beta+\gamma+\delta$.

For $\begin{cases} A.\alpha::B.\beta. \text{ and compounded.} \\ A+B.\alpha+\beta::(B.\beta)C.\gamma. \text{ And} \\ A+A+C.\alpha+\beta+\gamma::(C.\gamma)D.\delta. \&c. \end{cases}$

Alſo in continuall proportionalls, $\alpha.\beta::Z-\alpha.Z-a$. Wherefore $\alpha Z-\alpha q=\beta Z-\beta\alpha$. or $\beta Z-\alpha Z=\beta\alpha-\alpha q$. Hence by the way we may ſee how to finde the ſumme of all the termes $\div$, or of progreſſion Geometricall, by this rule $\frac{\beta\alpha-\alpha q}{\beta-\alpha}=Z$.

11. If the antecedents of many proportions bee equall; then as one of the antecedents is to the ſumme of his conſequents; ſo will the other antecedent bee to the ſumme of his. Let it be $A.B::\alpha.\beta$. and $A.C::\alpha.\gamma$. and $A.D::\alpha.\delta$. Then $A.B+C+D::\alpha.\beta+\gamma+\delta$. it appeareth out of the former demonſtration, the terms being alterndy placed.

12. If the conſequents of two Rations be equall, they are as their Antecedents: But if the Antecedents be equall, they are reciprocally as their conſequents.

$\frac{7}{9}.\frac{7}{9}::7.9$. And $\frac{1}{9}.\frac{1}{7}::7.9$.

13. If twice foure magnitudes be alike proportionall; then both their ſummes and alſo their differences ſhall be proportionall.

14. If foure proportionall magnitudes be multiplied or divided by foure other proportionall magnitudes; their products or quotients ſhall alſo be proportionall.

15 The Ration of the antecedent to the consequent is compounded, either of the Ration of the antecedent to a third, and of that third, to the consequent: or of the Ration of the third to the consequent, and of the antecedent to that third. As

7.9::× { 7.A. / A.9. } Also 7.9::× { A.9. / 9.A.

16. The Invention of the fourth proportionall terme in Astronomicall computations.

If 100000 be the first terme, the fourth proportionall is found by 5, Chap. IV. Case 3. As

100000. 80902::39875.32260.

If 100000 be the second or third terme, the fourth proportionall is found by 8. Chap. V.

137658. 100000::126225.91706.

17. The finding out of a part proportionall, by the difference of two numbers given, in the Canon of *Prosthapheresis*.

In the Prutenic tables, At Gr. 62 of the Anomalie of the Moons first Epicycle, the *Prosthaeresis* Gr. 4,1786 ablative. and the difference there Gr. 0,0433: how much of that difference shall be given, to the Anomalie, Gr: 62,5667? say

1. 0,0433::0,5667. 0,0245: by Chap: IV. Sect. V. Case 2. then 4,1786+0,0245=4,2031: which is the *Prosthaph:* corrected.

And contrariwise, if the Anomalie of the Moons first Epicicle, agreeing to the *Prosthaeresis* Gr

Gr. 4,2031. be required. The next leſſe in the Canon is Gr: 4,1786, anſwering to the Anomalie of Gr: 62. And the difference there is Gr: 0,0433. But 4,2031 — 4,1786 = 0,0245. ſay therefore

0,0433. 0,0245 :: 1. 0,5664 parts to be added to the Gr: 62. And the Anomalie ſought for, ſhall bee Gr: 62,5664.

18. The converſion of Sexageſimal into Decimals, and contrariwiſe of Decimal parts into Sexageſimals.

Sexageſimal parts, ſuppoſe 45, are turned into Decimals, by dividing them by 60. And contrarily Decimall parts, ſuppoſe 0,75, are turned into Sexageſimes, by multiplying them by 60. As

60. 45 :: 1. 0,75. } For
And 1. 0,75 :: 60. 45. }

Diviſion by 60 ſets backward the ſeparating line one place towards the left hand: And Multiplication by 60 ſets forward the ſeparating line one place towards the right hand: which rule is worth the obſervation.

But if there be more Sexageſimall *ſpecies* joyned to Integers, ſuppoſe 127° 32′ 00″ 09‴ 45ᴵᵛ: you may uſe this *compendium*. Under the Integers 127, ſet the Sexageſimal *ſpecies* in an oblique deſcent: Then beginning at the loweſt, divide every of them continually by 6: And ſetting the quotients over their heads,

joyne them, to the row next above, untill you come to the Integers.

```
      127|5333784722+
        32|002708333+
        ''00|1625
   6)   '''09|75   ×6
          ''45
```

And on the contrary, if Decimal parts be given, suppose 127|5333784729, you shall multiply them continually by 6, and write the products underneath, in euery row cutting off one place towards the right hand, that the oblque descent may bee compleated. View the example diligently.

Degrees of the Æquinoctiall with decimall parts, suppose Grad: 236|4276, are converted into decimall parts of a Day; dividing them by 360, that is 6×60: And contrarily, decimall parts of a Day, suppose 0|65674333, are turned into Degrees.

```
6)236|4276
60) 39|4046×6
    0|6567433:×60
```

multiplying them by 360, that is 60×6. View diligently the example.

Degrees of the Æquinoctiall with decimall parts, suppose Grad: 236|4276, are converted into houres, dividing them by 15, that is 3×5.

And

And contrariwiſe, Houres 3) 236|4276
with decimall parts, ſuppoſe 5) 78|8092*3
15|76184, are converted into 15|76184*5
Degrees, multiplying them by 15, that is 3*5.

Houres with decimall parts, ſuppoſe Ho: 15,76184, are converted into decimall parts of a Day, dividing them by 24, that is 4*6.

And contrarily, decimall parts 4) 15|76184
of a Day, ſuppoſe 0,6567433†, 6) 3|94046 *4
are converted into Houres multi- 0,6567433†.*6
plying them by 24, that is, 6*4.

A Collect Summe, ſuppoſe 191374, is converted into an Expanſe, dividing it continually by 60.

And contrariwiſe, the ſame Expanſe ſumme 53 09,34, is converted againe into a Collect, multiplying it continually by 60.

191374
60) 3189|3 *60
53|0

Here note, that if the Collect ſum be of units, namely 191374; the Expanſe wil be 53° 09' 34": that is, 53 ſexagene ſeconds, 9 Sexag: Primes, and 34 units. But if the Collect ſumme be of ſexageſimal ſeconds; the Expanſe will be 53°39'34".

19. The former Proportion was the equality of Rations: and is called Geometricall: There is ano-

ther proportion called Arithmeticall, which is the equality of the Differences : namely when as in foure termes, the ſame is the difference of the third and the fourth, which is of the firſt and the ſecond. As 7.4 & 12.9:or 7.7-3 and 12. 12-3 : are Arithmetically proportionall.

20. Wherefore of foure Arithmetically proportionalls, the ſumme of the extremes, is equall to the ſumme of the meanes. For 7+12-3=7-3+12.

21. And if, of three numbers given, the ſecond be added to the third, and the firſt taken out of the ſum, the Reſidue ſhall be the fourth Arithmetically proportionall. As if there be given 7, 4 and 12 : then will 12+4-7=9 the fourth number deſired.

22. There is alſo a continued Arithmeticall Proportion, or Progreſſion; when all the termes from the firſt, doe continually riſe with the ſame difference. As 4,7,10,13,16,19 &c. The common difference of them all is 3. For in this *ſeries*, the firſt terme (and as it were the root) is 4 : the ſecond conſiſts of the firſt and one difference; the third conſiſts of the firſt and two differences. And generally every terme conſiſteth of the firſt and a ſumme of differences, whoſe number is leſſe by one than the number of the termes. For example, the thirteenth terme ſhall be made up of the firſt and twelve differences, whoſe ſumme is 36. therefore 4+36, that is, 40, is the thirteenth terme.

23. If in Arithmeticall Progreſſion the firſt terme

be

be added to the laſt, and the ſumme drawne into the number of the termes; the product will be the double ſumme of the whole Progreſſion. Namely 40+4 into 3=572, which is the ſumme doubled.

24. If over a *ſeries* of the termes in Geometricall Progreſſion, be ſet, for Indices, a *ſeries* of any termes in Arithmeticall progreſſion: to every foure numbers taken in Arithmeticall proportion, there will anſwer foure numbers Geometrically proportionall.

Indices, 6. 8. 10. 12. 14. 16. 18. 20.
Termes, 5. 15. 45. 135. 405. 1215. 3645. 10935.

Becauſe 10+16-6=20. It will be $\frac{45*1215}{5}=10935$. From hence it appeareth how to finde out any terme in Geometricall Progreſſion.

25. There is alſo a third Proportion, called Muſicall; when in foure numbers, as the Firſt is to the 4th, ſo is the difference of the firſt and the ſecond, to the difference of the third and the fourth: as 5, 8, 12, 30, are Muſically proportionall. Becauſe 5.30::8-5.30-12::3.18. Likewiſe in *Species*, A, M, N, E. Let it be A, E::M-A.E-N. Wherefore AE-AN=ME-AE. Theſe termes being rightly ordered, the rule will be $\frac{AN}{2A-M}=E$. And $\frac{EM}{2E-N}=A$. In words thus, If the rectangle contained under the firſt and third terms, be divided by the exceſſe of the firſt doubled above the ſecond; the quotient will be the fourth terme in Muſicall proportion. Wherefore it behoveth the termes

termes ſo to be given, that the firſt doubled may exceed the ſecond.

Chap. VII.

OF THE GREATEST COMMON MEASURE: whereby numbers given are reduced to the leaſt termes of the ſame Ration.

1. THE Greateſt common meaſure of two numbers is found out, by the continuall diviſion of the greater by the leſſe, and againe of the diviſor by the remainer: for that diviſor, which firſt meaſureth his dividuall without any remaines, ſhall be the greateſt common meaſure of both the numbers given. As of the numbers 899 and 744 the greateſt common meaſure ſhall be found 31.

31 124 155

31) 124)155)744)899(124

124 124 620 744

2. Numbers are reduced to the leaſt termes of the ſame ration, by dividing them both by their greateſt common meaſure, as 899 and 744 are reduced to 29 and 24, which are the leaſt termes in the ſame ration, both of them being divided by 31 their greateſt common meaſure. So $\frac{3Aq}{6A}$ are reduced to $\frac{A}{2}$, dividing both termes by 3A. And $\frac{4Acc}{6Aqq}$ are reduced to

2Aq

$\frac{2Aq}{3}$ dividing both by 2Aqq. Likewiſe $\frac{BA}{B}$ is reduced to A, dividing both by B. For that which Multiplication makes up, Diviſion diſſolves.

3. Wherefore, if the greateſt common meaſure of two numbers, be 1. Thoſe numbers are called Prime, one to another: And they are the leaſt numbers in the ſame ration. As 29 and 24.

4. If a number bee prime to both the numbers multiplied, it will be prime to the product: From hence the worke of proportion 1 3 2 5 may oftentimes bee effected with more eaſe, as in example. 22. 8 :: 15. 10

5. But remember diligently, *As often as any fraction, or Ration, is propounded, that you firſt reduce it to the leaſt termes*; As $\frac{141}{235}$ to $\frac{24}{25}$.

Chap. VIII.

OF PARTS: which are alſo called fractions or broken numbers.

1. AN unit (or any Integer whatſoever) may in minde be divided into never ſo many equall parts: which parts take their denomination from that number of them, which the unit containeth: as if an unit be conceived to be divided into two equal parts, they are called Seconds; if into three, Thirds: and ſo of the reſt.

2. Parts are written with two termes, having a ſmall

ſmall line drawne between them : the lower of which denoteth the unit divided into ſo many equall parts; and is called the Denominator. But the upper ſhewes how many of thoſe parts are ſignified : and is therefore called the Numerator.

As $\frac{4}{5}$ Numerator / Denominator { and ſignifie foure fifth parts, or foure parts of one integer divided into five.

3. Therefore what Ration the *Numerator hath to the Denominator, the ſame hath the quantity ſignified to an unit.*

$4.5 :: \frac{4}{5}.1.$ R.S:: $\frac{R}{S}.1.$

4. And becauſe every Ration may be expreſſed by innumerable ſeverall termes (the greateſt of which can never be given) in the ſame habitude to one another; it followes, that the ſame parts may be deſigned not only by the ſame numbers, but alſo by infinit others, as the Portion called, *Quincunx* is not only ſignified by $\frac{5}{12}$ (which are the leaſt termes in that Ration) but alſo by $\frac{10}{24}, \frac{20}{48}, \frac{40}{96}$, &c. and by ſo many numbers as can be made by multiplying 5 and 12 into any other number by Sect. 2. Chap. 6.

5. Wherefore the termes of equall parts or fractions are proportionall, and contrariwiſe.

6. Likewiſe, if the Numerator of the parts be leſſe then the Denominator, the parts are leſſe then an unit: if equall, they ſignifie an unit : and if greater, the parts exceed an unit in the ſame Ration, as the Numerator

rator exceedeth the Denominator. Now they are reduced to units, dividing the Numerator by the Denominator: as $\frac{15}{7}$ become $4\frac{1}{7}$. Likewise $\frac{CR+SA}{R}$ is $C+\frac{SA}{R}$. And on the contrary, units are resolved into parts of any *Kinde*, multiplying the units by the Denominator of those same parts; as 1 may be made $\frac{3}{3}$, or $\frac{4}{4}$ &c: and $4\frac{1}{7}$ will be made $\frac{28+3}{7}$ that is $\frac{31}{7}$. Likewise $C+\frac{SA}{R}$ will become $\frac{CR+SA}{R}$.

Chap. IX.

OF THE ADDITION AND SUBDUCTION OF PARTS.

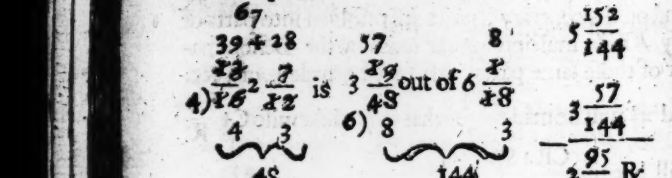
1. IF the parts proposed be of divers *species* first reduce them to the same denomination, dividing the Denominators by their greatest common measure; and multiplying the termes by each others Quotients: Then make your Addition or Subduction in the Numerators of those new parts having the same denomination. And lastly, write that common Denominator under the Summe or the Difference.

2. And if Integers be mixed with the parts, they are notwithstanding to be numbred severally apart.

For Example sake.

Out of $6\frac{1}{16}$ take $\frac{13}{16}$ and $2\frac{7}{13}$. First adde $\frac{13}{16}$ and $2\frac{7}{13}$, and

and they shall be 2 and $\frac{39+28}{48}$ or $\frac{67}{48}$, that is $3\frac{19}{48}$: which being taken out of $6\frac{1}{12}$, there shall remaine $2\frac{25}{144}$. As in Example.



To $\frac{A}{B}$ adde Z, Summe $\frac{A+ZB}{B}$

Out of $\frac{A}{B}$ take $\frac{B}{C}$, Remains $\frac{CA-Bq}{BC}$

Chap. X.

Of the Multiplication and Division of Parts.

1. Multiplication compareth the unlike termes, (that is reduceth them to the least) and multiplieth the like termes.

2. Division compareth the like termes, and multiplieth the unlike.

3. And

3. And if integers be mixed with the parts, the integers are to be resolved into parts.

$\frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{16}}}$ into $\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{27}}}$ make $\frac{5}{12}$ | $\frac{\overset{4}{\cancel{8}}}{\cancel{9}}$ into $\frac{5}{\underset{3}{\cancel{6}}}$ makes $\frac{20}{27}$ |

$\frac{5}{1}$ into $\cancel{3}\frac{\overset{13}{\cancel{1}}}{4}$ makes $\frac{65}{4}$ ($16\frac{1}{4}$

$\frac{A}{B}$ into B maketh A | $\frac{A}{B}$ into Z makes $\frac{ZA}{B}$ |

$\frac{A}{B}$ into $\frac{ZA}{C}$ makes $\frac{ZAq}{BC}$

Examples of Division.

$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{16}}}$) $\frac{\overset{5}{\cancel{15}}}{\underset{7}{\cancel{28}}}$ ($\frac{20}{21}$ | $\frac{8}{\underset{3}{\cancel{15}}}$) $\cancel{7}\frac{\overset{37}{\cancel{2}}}{\underset{1}{\cancel{5}}}$ ($\frac{111}{8}$ ($13\frac{7}{8}$ | $\frac{\overset{1}{\cancel{3}}}{4}$) $\frac{\overset{3}{\cancel{9}}}{1}$ ($\frac{12}{1}$

$\frac{D}{1}$) $\frac{Aq}{B}$ ($\frac{Aq}{DB}$ | $\frac{A}{D}$) $\frac{BC}{1}$ ($\frac{BCD}{A}$ | $\frac{A}{B}$) $\frac{BC}{1}$ ($\frac{BqC}{A}$

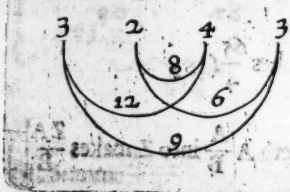
$\frac{B}{A}$) $\frac{BC}{1}$ ($\frac{CA}{1}$ | $\frac{Ac}{C}$) $\frac{Bc}{D}$ ($\frac{BcC}{DAc}$

4. What

4. What number is $\frac{2}{3}$ of 21? Multiply 21 by $\frac{3}{2}$. For $1.\frac{2}{3}::21.6.$ or $7.2::21.6.$

5. Of what number doth 6 contain $\frac{2}{7}$? Divide 6 by $\frac{2}{7}$. For $\frac{2}{7}.1::6.21.$ or $2.7::6.21.$

6. Amongst ancient Writers of Musique, the termes to be multiplied either in the continuation or imminution of Rations, are knit together in this manner.



7. The Continuation of Rations is performed by multiplying them, as if they were fractions. Continue the Rations 3 to 2, and 4 to 3: It is all one as if it were said, multiply $\frac{3}{2}$ into $\frac{4}{3}$, the product is $\frac{12}{6}$ which is a double Ration. Wherefore the Ration of one and a halfe continuated with the Ration of one and one third, maketh a double: or as Musitians speake, Of a *diapente*, and a *diatessaron*, is made a *diapason*.

8. The Imminution of Rations is performed by Division: as if from the Ration 2 to 1, be to be taken the Ration 4 to 3: It is all one as if you were desired to divide $\frac{2}{1}$ by $\frac{4}{3}$; and there shall remaine $\frac{8}{9}$. For $\frac{3}{4})\frac{2}{1}(\frac{8}{9}$ the Ration of one and one eight; which is the measure of a whole Tone. Whence the Musitians say. That the difference between *diapente* and *diatessaron* is a Tone.

Tone. As in the line or chorde divided into twelve parts is ſeen.

CHAP. XI.

Some eaſie Examples, by which, what hath hitherto been delivered, is rendred familiar; and the way made to Analyticall Æquation.

1. YOU muſt firſt of all know, that both for brevity ſake, and helping the imagination, I uſe, almoſt every where, theſe notes or ſymbols of words. A and E, ſignifie two numbers or magnitudes; of which A, for the moſt part, is the Greater; and E the Leſſe. Æ ſignifies the Rectangle contained under them. Z the Summe. X the Difference. Zq the Quadrat of the Summe. Xq the Quadrat of the Difference. Z the Sum of the quadrats. X the difference of the quadrats. Z the Sum of the Cubes. X the difference of the cubes. A,M,E, Three continually Proportionalls. A,M,N,E, Foure. Q:C:QQ:Qc:&c. ſet before Magnitudes included between two points on both ſides, ſignifie the reſpective *Poteſtates* of that ſort. √ denoteth the Root or ſide of a ſimple *poteſtas*, if there intercede not two points: but if the *poteſtas* be included between the two points on either ſide, it ſignifies his univerſall Root or ſide, which alſo is wont to be deſcribed by the letter b or r, as √b is the ſide of a Binomiall and

√r the Side of a Reſiduall or *Apotome*. = is a Note of Equality.

2. There are two numbers or magnitudes, whereof the greater is A, the leſſer E: what is the Summe of them? what is the Difference? what the Rectangle under them? what the Summe of the Quadrats? what the Differeнce of the Quadrats? what's the Summe of their Summe and Difference? what the Difference of their Summe and Difference? what the Rectangle of their Summe and Difference? what the Quadrat of the Summe of them? what the Quadrat of the Difference? what the Summe of the Quadrats of the Summe and of the Difference? what the Difference of the Quadrats of the Summe and the Difference? what the Quadrat of the Rectangle?

Theſe queſtions are thus reſolved.

Z is A+E. X is A−E. Æ is AE.

Z=Aq+Eq. X=Aq−Eq.

Z+X=2A. Z−X=2E.

½Z+½X=A. ½Z−½X=E.

ZX=Aq−Eq=X.

Zq=Aq+2AE+Eq. Xq=Aq−2AE+Eq.

Zq+Xq=2Aq+2Eq. Zq−Xq=4AE. ¼Zq−¼Xq=AE

Æq=Aq Eq.

3. There are two numbers or magnitudes, whoſe Summe is Z, and the Greater of them is A, what is the Leſſer? what their Difference? what the Rectangle under them? what the Summe of the Quadrats?

drats? what the Difference of the Quadrats?
E=Z-A. X=2A-Z. Æ=ZA-Aq.
Z=Zq-2ZA+2Aq. X=2ZA-Zq.

But if the leſſer of them be put E.
A=Z-E. X=Z-2E. Æ=ZE-Eq.
Z=Zq-2ZE+2Eq. X=Zq-2ZE.

4. There be two numbers or magnitudes, whoſe Difference is X, and the Greater of them is put A. What is the Leſſer? what their Difference? what Summe their Rectangle under them? what the Summe of the Quadrats? what the Difference of the Quadrats?
E=A-X. Z=2A-X. Æ=Aq-AX.
Z=2Aq-2XA+Xq. X=2XA-Xq.

But if the leſſer of them be put E.
A=E+X. Z=2E+X. Æ=Eq+XE
Z=2Eq+2XE+Xq. X=2XE+Xq.

5. There are two numbers or magnitudes, of which the greater to the leſſer hath the Ration of R. to S. And for the greater of them is put A. What is the leſſer? what's their Summe? what their Difference? what is the Rectangle under them? what the Summe of the Quadrats? what the Difference of the Quadrats?

$$E=\frac{SA}{R}\ Z=\frac{RA+SA}{R}\ X=\frac{RA-SA}{R}$$

$$Æ=\frac{SAq}{R}\ Z=\frac{RqAq+SqAq}{Rq}\ X=\frac{RqAq-SqAq}{Rq}$$

But if the leſſer of them be put E.

$$A=\frac{RE}{S}.\ Z=\frac{RE+SE}{S}.\ X=\frac{RE-SE}{S}.$$

$$Æ=\frac{REq}{S}.\ Z=\frac{RqEq+SqEq}{Sq}.\ X=\frac{RqEq-SqEq}{Sq}.$$

6. There are two numbers or magnitudes, whoſe rectangle is Æ; and the Greater of them is put A. What is the Leſſer? what their Summe? what their Difference? what is the Summe of the Quadrats? what the Difference of the Quadrats?

$$E=\frac{Æ}{A}.\ Z=\frac{Aq+Æ}{A}.\ X=\frac{Aq-Æ}{A}.$$

$$Z=\frac{Aqq+Æq}{Aq}.\ X=\frac{Aqq-Æq}{Aq}.$$

But if the leſſer of them be put E.

$$A=\frac{Æ}{E}.\ Z=\frac{Æ+Eq}{E}.\ X=\frac{Æ-Eq}{E}.$$

$$Z=\frac{Æq+Eqq}{Eq}.\ X=\frac{Æq-Eqq}{Eq}.$$

7. And out of theſe compared many equalities do ariſe, we will take examples in the Summe and Difference.

$$Z=A+E=2A-X=3E+X=\frac{Aq+Æ}{A}=\frac{Æ+Eq}{E}\ \&c.$$

$$X=A-E=2A-Z=Z-2E=\frac{Aq-Æ}{A}=\frac{Æ-Eq}{E}\ \&c.$$

And in this manner compariſons may be framed in the reſt.

CHAP.

CHAP. XII.

OF THE GENESIS AND ANALYSIS OF POTESTATES.

1. BEcauſe that all things are reſolved into the ſame parts, of which they are compoſed: it is neceſſary firſt of all to know, of what parts every *Poteſtas* conſiſteth. Now the *Poteſtates* are made of a Root or Side multiplied into it ſelfe, ſome certaine times. For a Side drawne into it ſelfe makes a Quadrat. A Quadrat drawn into its Side, maketh a Cube. A Cube drawne into its Side, makes a Quadrato-quadrat, which is the *quartaine poteſtas* |4|: this againe drawne into its Side, makes a quadrato-cube, *viz.* the *quintane* |5|: and ſo going on ſtill further, there are made *Poteſtates ſextane* |6|, *ſeptimane* |7|, *octavane* |8|, *nonane* |9|, *decimane* |10|, and the reſt, acccording to the number of their dimenſions, of which they are compounded.

2. Wherefore the procreation of *Poteſtates*, from a ſingle Root, which conſiſts of one figure or note, hath no difficulty at all.

THE FORMER TABLE OF POTESTATES FROM A SINGLE ROOT.

L N	2 q	3 c	4 qq	5 qc	6 cc	7 qqc	8 qcc
1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256
3	9	27	81	243	729	2187	6561
4	16	64	256	1024	4096	16384	63536
5	25	125	625	3125	15625	78125	390625
6	36	216	1296	7776	46656	279936	1679616
7	49	343	2401	16087	117649	823543	5764801
8	64	512	4096	32768	262144	2097152	16777216
9	81	729	6581	59049	531441	4782969	43046721

3. But *Poteſtates* which ſpring from a Root of two Notes, doe ariſe after this manner.

Geneſis of *Poteſtates* from a Binomial Root.

A+E
A+E
Aq+AE
+AE+Eq
Aq+2AE+Eq. Quadrat.
A+E
Ac+2AqE+AEq
+AqE+2AEq+Ec
Ac+3AqE+3AEq+Ec. Cube.
A+E
Aqq+3AcE+3AqEq+AEc
+AcE+3AqEq+3AEc+Eqq
Aqq+4AcE+6AqEq+4AEc+Eqq. Quadrato-quadrat.
A+E &c.

4. And by this Artifice may be framed a Table of *Poteſtates* aſcending in the ſcale from a Binomiall Root; which may be named the LATTER.

the Quadrats will interpose[illegible]

plements,

Side or Number	1	2	3	4	5	6	7	8	9	10
A		1	1	1	1	1	1	1	1	1
E		2AE	3AEq	6AEqq	5AEqq	6AEqc	7AEqqc	8AEqqc	9AEqcc	10AEccc
Aq		3Aq	3AqE	4AqE	5AqqE	15AqqEq	21AqqEq	28AqqEc	36AqqEq	45AqqEc
Eq		Eq	3AEq	6AqEq	10AqEq	15AqEqq	35AqqEc	56AqEqc	84AcEc	120AqqEc
Ac			1	1	1	1	1	1	1	1
Ec			Ec	4AEc	5AEqc	6AEcc	7AEqqc	8AEqqc	9AEqcc	10AEccc
Aqq				4AcE	10AcEq	20AcEc	35AcEqq	56AcEqc	126AqqEqc	210AqqEc
Eqq				Eqq	5AEqq	15AqEqq	21AqEqc	28AqEcc	36AqEqqc	45AqEqc
Eqc					Eqc	6AEqc	7AEcc	8AEqqc	9AEqcc	10AEccc
Acc						1	1	1	1	1
Ecc						Ecc	Eqqc	Eqqc	Eqcc	Eqqcc
Aqqc							1	1	1	1
Eqqc							Eqqc	8AEqqc	9AEqcc	10AEccc
Aqcc								1	1	1
Eqcc								Eqcc	Eqcc	Eqqcc
Accc									1	1
Eccc									Eccc	Eqccc
Aqqqc										1
Eqqqc										Eqqcc

5. Every immediate *ſpecies* of each row is made up of the two *ſpecies* of the former row; next unto it on both ſides. Namely the *poteſtas* A, of the upper *ſpecies*, and the *poteſtas* E, of the lower. The number alſo to be ſet before it, is aggregated of both their numbers adfixed. Wherefore this Table may be eaſily continued further at pleaſure.

6. In this Table, the two extreame *poteſtates* of the ſeverall *genus* or kinds, are *diagonals*: and the intermediat *ſpecies* are Complements, to which are affixed *unciæ* or numerall figures, ſhewing the number of Complements to bee taken in the conſtitution of each *Poteſtas*. Now all the Complements with the *Poteſtas* E, may not unfitly be termed the *Gnomon*.

7. Out of this Table alſo it appeareth, that the quadrat of a Root of two Notes or *Nomina* conſiſteth of the *diagonal Quadrats* of both Notes; and a double Rectangle contained under the Notes: But a Cube conſiſteth of the *diagonal* Cubes, and a triple Solid under the Quadrat of the greater Note, and the leſſer Note; and alſo a triple Solid under the greater Note, and the Quadrat of the leſſer. Which in like manner is to be pronounced alſo of the other *Poteſtates*.

8. Moreover this Table, full of excellent myſteries, ſheweth, in a numerous *Poteſtas*, the ſeats, both of the *ſingle or diagonal Poteſtates*, and of every *ſpecies* of Complements. For ſeeing, there is but one *ſpecies* between the two quadrats; the ſeates of the Quadrats will interpoſe only one place for Complements,

plements, and ſeeing between the two Cubes, there are two *ſpecies* of Complements, the Seats of the Cubes will leave two places, to be diſtributed to their Complements in due order.

Chap. XIII.

Theſe things being premiſed, let us come to THE GENESIS OF POTESATES.

1. Let the *Geneſis* of a Quadrat from the Side 57 be propounded. The greater Note A, is 5, the leſſer E, is 7. Write 5 and 7 intermitting the ſpace of one Grade or place: and draw a line under them. Under 5, ſet his Quadrat 25: and under 7, his, 49. then double 5, and multiply it by 7, and the Rectangle doubled will be 70, to be ſet in the Intermedial place. Adde them altogether in their proper places: and the Summe will be 3249, for the Quadrat of the Side 57 deſired.

5	7	
25		Aq
7	0	2AE } gnomō
	49	Eq }
32	49	

2. Againe let the *Geneſis* of a Cube from the Side 57 be propounded. Write 5 and 7 intermitting the ſpace of two Grades: and draw a line under them: under 5, ſet his Cube 125: and under 7, his, 343. Then triple the Quadrat of 5, and multiply it by 7, and the greater Solid Tri-

5	7	
125		Ac
52	5	3AqE }
7	35	3AEq } gnomon
	345	Eq }
185	193	

pled

pled will be 525, to be put in the former Intermedial place: likewiſe triple 5, and ſo multiply it by 49, the Quadrat of 7, and the leſſer Solid Tripled will be 735, to be put in the latter Intermedial place: Adde all together, every of them in their owne places: the Samme will be 185193 for the Cube of the Side 57 deſired.

3. If the Side propounded conſiſt of more figures, as 57209: the *Poteſtas* of the two former figures 57, is firſt of all to be ſought: then taking 57 for A, and 2 the next following figure for E: ſeeke its *Poteſtas* in the ſame order of the Table, which is before ſhewed: which alſo is to be done through all the reſt of the figures ſeverally.

5	7	2	0	9	Root
25	Aq				
70	2AE	} Gnomon			
49	Eq	}			
32	49	Aq			
	228	2AE	} Gnomon		
	4	Eq	}		
32	71	84	00	Aq	
	1	02	96	0 2AE	} Gnomon
				81	Eq }
32	72	86	96	81	Quadrat.

5	7	2	0	9	Root
125	Ac				
	525	3AqE			} Gnomon
	735	3AEq			} Gnomon
	343	Ec			} Gnomon
185	193	Ac			
	1949	4	3AqE		} Gnomon
	684		3AEq		} Gnomon
	8		Ec		} Gnomon
187	149	248	000	Ac	
	88	339	680	0 3AEq	} Gnomon
		13	899	60 3AEq	} Gnomon
			729	Ec	} Gnomon
187	237	601	580	329	Cube.

4. Out of what hath already bene declared, it will not be difficult to beget alſo all the other *Poteſtates* of the ſuperiour kinds; ſo that in their geniture the *Geneſis* of all the *Poteſtates* aſcending unto them, bee inſtituted: as you now ſee done in the *Geneſis* of a Cube.

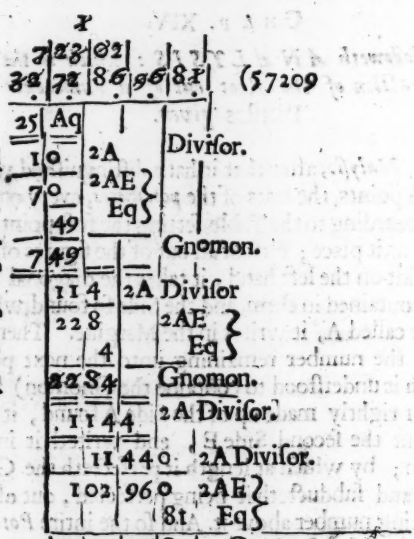
CHAP. XIV.

Here followeth ANALYSIS: which is the extraction of the Root out of a numerous Poteſtas *given.*

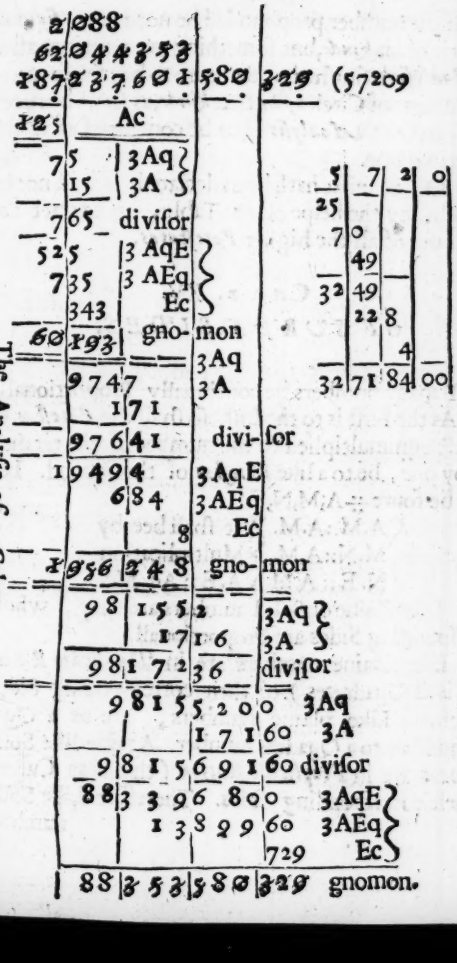
1. *Analyſis*, after that it hath diſtinguiſhed, with points, the ſeats of the *poteſtates*, every one in *kinde* according to the Table, ſetting the firſt point under the unit place; Firſt of all out of the figures of the firſt point on the left hand, it takes the diagonal *Poteſtas* contained in them, and the Side ſo found, which may be called A, it writes in the Margine. Then dividing the number remaining unto the next point (which is underſtood to containe the Gnomon) by a Diviſor rightly made up of the Side A found, it ſeeketh out the ſecond Side E, and writeth it in the Margin; by which at length it perfecteth the Gnomon, and ſubducteth it being perfected, out of the remaining number above it. And ſo the intire *Poteſtas* of the two firſt ſingle Sides contained in the two firſt Points, being taken out, there will remaine, unto the third Point, a Gnomon for the third Side, in like manner to be extracted.

The

The Analysis of a Quadrat.



The



The Analysis of a Cube.

2. If the number propounded be not a true *figurat* number of his *kinde*, but ſomething yet remaines after the *Analyſis* is finiſhed: there are to bee ſet ſo many Punctations of Circles, for that *kinde*, as ſhall be needfull: and the *Analyſis* is to be continued after the ſeparating line.

3. Out of what hath been declared, it will not be difficult, by the helpe of the Table, to extract the Roots out of all the higher *Poteſtates*.

CHAP. XV.

OF SURDE SIDES.

8. IF many numbers be continually Proportionall; As the Firſt is to the Laſt, ſo ſhall the *Poteſtas* of the firſt æquimultiplied to the number of the termes, leſſe by one, be to a like *Poteſtas* of the Second. Let there be foure ∴ A, M, N, E.

Becauſe { A.M. :: A.M. / M.N :: A.M. / N.E :: A.M } It ſhall bee by Multiplication A.E :: Ac. Mc.

2. Like Plaine or Solid numbers are they, whoſe correſponding Sides are Proportionall.

3. Like Plaine numbers are in *Duplicate Ration* (that is as Quadrates) of their correſponding Sides. Therefore Like plaine numbers, are as a Quadrat number to a Quadrat number. Againe like Solid numbers are in *Triplicate Ration* (that is as Cubes) of their correſponding Sides. Therefore Like Solid numbers

numbers are as a Cubicall number to a Cubicall number.

4. And generally all like Figurate numbers, of more dimenſions, are in Ration of their correſponding Sides multiplied equally to the number of their dimenſions, of which they are compounded.

Let the dimenſions be foure { A.B.C.D. of the one, / E.F.G.H. of the other } In the *Ratio* of R. to S.

Becauſe { A.E :: R.S / B.F :: R.S / C.G :: R.S / D.H :: R.S } It ſhall be by Multiplication. ABCD.EFGH :: Rqq.Sqq.

5. If a number be not a true Figurate number of his *kinde*, the Side thereof is called Surde: and is thus noted, √q6, √c4, √qq20, √qc13: that is, the Side of the Quadrat 6, the Side of the Cube 4, the Side of the Quadrato-quadrat 20, the Side of the Quadrato-cube 13. &c.

6. Commenſurable Surde Sides are thoſe, whoſe numbers being reduced to the leaſt termes, become true Figurate numbers of their *Kinde*; and are therefore as a number to a number. As √q12 and √q147 reduced to the leaſt termes, by √q3, the greateſt common meaſure of them both, become √q4 and √q49. that is 2 and 7. Wherefore ſeeing that √q12 and √q147, are as 2 to 7, they ſhall be commenſurable. Thus alſo √c40 and √c1715, are as 2 to 7, becauſe being divided by their greateſt common meaſure

ſure $\sqrt{c}\,5$, they become $\sqrt{c}\,8$ and $\sqrt{c}\,343$. And therefore are commenſurable.

7. Commenſurable Surde Sides are added or Subducted, if the ***homogene Poteſtas*** of the Summe or Difference of their like numbers found, be drawn into their common meaſure. As $\sqrt{q}\,147+\sqrt{q}\,12$ is $\sqrt{q}\,243$, that is, the Side of the Quadrat made of $7+2$ (namely 9) drawne into $\sqrt{q}\,3$ their greateſt common meaſure. And $\sqrt{q}\,147-\sqrt{q}\,12$ is $\sqrt{q}\,75$, that is, the ſide of a Quadrat made of $7-2$ (*viz.* 25) drawne alſo into $\sqrt{q}\,3$.

Likewiſe $\sqrt{c}\,1715+\sqrt{c}\,40$ is $\sqrt{c}\,3645$, that is, the Side of a Cube made of $7+2$ (namely 729) drawne into $\sqrt{c}\,5$ their greateſt common meaſure. And $\sqrt{c}\,1715-\sqrt{c}\,40$ is $\sqrt{c}\,625$; that is, the ſide of a Cube made of $7-2$ (*viz.* 125) drawne alſo into $\sqrt{c}\,5$.

The Operation of Addition and Subduction is thus:

$$\sqrt{q}\,3)\begin{array}{l}\sqrt{q}\,147\,(\sqrt{q}\,49.7.\\ \sqrt{q}\,12\,(\sqrt{q}\,4.2.\end{array}\qquad \sqrt{c}\,5(\begin{array}{l}\sqrt{c}\,1715\,(\sqrt{c}\,343.7.\\ \sqrt{c}\,40\,(\sqrt{c}\,8.\,2\end{array}$$

$$\begin{array}{ll}\sqrt{q}\,243.\ \sqrt{q}\,81.\,9 & \text{Summe}\ \sqrt{c}\,3645.\ \sqrt{c}\,729.9.\\ \sqrt{q}\,75.\ \sqrt{q}\,25.5 & \text{Difference}\ \sqrt{c}\,625.\ \sqrt{c}\,125.5.\end{array}$$

$$\begin{array}{rl}& \sqrt{q}\,12+\sqrt{q}\,\tfrac{27}{4}\\ \text{Or} & \sqrt{q}\,\tfrac{48}{4}+\sqrt{q}\,\tfrac{27}{4}\\ \sqrt{q}\,3) & \sqrt{q}\,48\,(\sqrt{q}\,16.4\\ & \sqrt{q}\,27\,(\sqrt{q}\,9.\,3\\ & \sqrt{q}\,\tfrac{147}{4}:\ \sqrt{q}\,49.7\end{array}\qquad \begin{array}{rl}& \sqrt{q}\,\tfrac{245}{15}+\sqrt{q}\,\tfrac{5}{13}\\ \sqrt{q}\,5) & \sqrt{q}\,245\,(\sqrt{q}\,49.7\\ & \sqrt{q}\,5\,(\sqrt{}\,1.\,1\\ & \sqrt{q}\,\tfrac{320}{13}:\ \sqrt{q}\,64.8\\ & \sqrt{q}\,\tfrac{180}{15}:\ \sqrt{q}\,36.6\end{array}$$

8. But

8. But incommenſurable Surde Sides and Heterogeneal are added or ſubducted by the ſignes + or −. as √q7+√q4. and √c10−√c5.

9. If a Figurate number be multiplied by an Homogeneal Figurate number, the Product ſhall be a Figurate number of the ſame *kinde*, whoſe Side is equall to the Product of the Sides of the numbers multiplied. And if a Figurate number be divided by an Homogeneal Figurate number, the Quotient ſhall be a Figurate number of the ſame *kinde*, whoſe Side is equall to the Quotient of the Side of the greater number divided by the Side of the leſſer. As the product of the Cubicall numbers 343 and 27 is 9261 a Cubicall number alſo, whoſe Side is 7*3. Likewiſe $\sqrt{q}\frac{AqEq}{Bq}$ is $\frac{AE}{B}$.

10. Wherefore Multiplication and Diviſion of Homogeneal Surde Sides, gives alſo a Surde Homogeneal Side. As √q7 into √q3 is √q21. And √q7) √q21 (√q3. Or $\sqrt{q}\frac{21}{7}$ is √q3. Likewiſe √qA into √qE is √q AE. And √qA) √qAE (√qE: Or $\sqrt{q}\frac{AE}{A}$ is √qE.

11. But Heterogeneal Sides are neither multiplied nor divided, unleſſe they be firſt reduced to the ſame *kinde*. Which is effected by dividing the Indices of both the *Poteſtates* propoſed by their greateſt common meaſure: and multiplying the Indices by each others Quotients; and the *Poteſtates* themſelves into

the *species* denominated by each others Quotient. As if there be given to be Multiplied, or Divided $\sqrt{qq}$ 10 and $\sqrt{cc}$ 7. First Reduce them to $\sqrt{cccc}$ 1000, and $\sqrt{cccc}$ 49, Cubing 10, and Quadrating 7, then make your Multiplication or Division. Thus also $\sqrt{qq}$ A and $\sqrt{cc}$ Bq are Reduced to $\sqrt{cccc}$ Ac, and $\sqrt{cccc}$ Bqq. As will more plainly appeare by the practise which is here set downe.

$$\begin{array}{lllll} & \sqrt{\boxed{12}}\,1000 & \sqrt{\boxed{12}}\,49 & \sqrt{\boxed{12}}\,Ac & \sqrt{\boxed{12}}\,Bqq \\ \boxed{2}) & \sqrt{\boxed{4}}\,10 & \sqrt{\boxed{6}}\,7 & \boxed{2})\sqrt{\boxed{4}}\,A & \sqrt{\boxed{6}}\,Bq \\ & \boxed{2} & \boxed{3} & \boxed{2} & \boxed{3} \end{array}$$

Againe if $\sqrt{c}$ 32 be to be doubled, or multiplied by 2: in stead of 2 take $\sqrt{c}$ 8: and by it multiply $\sqrt{c}$ 32. And there will be made $\sqrt{c}$ 256; æquivalent to twice $\sqrt{c}$ 32.

Likewise if $\sqrt{c}$ 32 be to be Halfed, or Divided by 2: instead of 2 take $\sqrt{c}$ 8, and by it divide $\sqrt{c}$ 32; and there will arise $\sqrt{c}\,\frac{32}{8}$; that is $\sqrt{c}$ 4, æquivalent to $\frac{1}{2}$ $\sqrt{c}$ 32.

Thus also $\frac{22}{7}$ $\sqrt{q}$Aq, will be made $\sqrt{q}\,\frac{484}{49}$ Aq, that is $\frac{22}{7}$A.

12. If the Side of a *Potestas* be to be multiplied according to the exigencie of its owne *species*; blot out the Laterall Note of the *species*: As Q: $\sqrt{q}$ 64, or C: $\sqrt{c}$ 64, is 64.

13. And if the Side of a *Potestas*, whose Index is a compounded number, be to be multiplied accor-

ding

ding to the exigencie of either of the compounding *ſpecies*; the Side of the other *ſpecies* ſhall only be prefixed to the Speciall number: As Q:√cc 64 is √c 64. and C:√cc 64 is √q 64. For √cc is $\sqrt{|2\times 3|}$.

14. If a Magnitude of many *nomina* be drawne into it ſelfe, with one of his ſignes changed; one *nomen* ſhall be expunged. 3+√5+√2 into 3+√5-√2. maketh 10+√180.

Chap. XVI.

OF ÆQUATION. And Queſtions to bee reſolved by Æquation.

1. WHenſoever any Probleme or Queſtion is propounded; ſuppoſe that done, which is deſired: and having uſed a fit Ratiocination, for the Magnitude ſought, put A, or ſome other vowel; and for the Magnitudes given, put Conſonants; that thereby the Magnitudes given, may the more eaſily be diſtinguiſhed from thoſe that are uncertaine.

2. Then let both the given and ſought Magnitudes, according to the Queſtion, be tranſlated and compared, by Adding, Subducting, Multiplying, and Dividing them: untill at length there be found ſomething equall, either to the Magnitude, whereof the Queſtion is, or to ſome *Poteſtas* thereof, to which it ſhall adſcend.

3. And becauſe that almoſt in every Æquation,

when it first begins to shine out of the coverts of the Question, things known are confounded with things unknowne: its termes are so to be ordered, that those things which are given in measure, may make one part of the Æquation; and those which being unknowne, are sought, the other. Which by what Artifice it is to be done, shall be shewed by the five following Rules.

4. First, if the Magnitude sought, or any degree thereof, be in a Fraction: *Reduce* all the Magnitudes to one Denomination: that omitting that common Denominator, the Æquation may be valued in the Numerator only. As $A \S A - C = \frac{Aq+Bq}{D} + B + C$: shall be DA-DC=Aq+Bq+DB+DC.

5. Secondly, if those things which are given in measure, be mingled with those which are sought, *Let there be a Transposition of the Magnitudes, from the one side to the other, under a contrary signe.* As DA-DC=Aq+Bq+DB+DC. And the Magnitudes DC and Aq being transposed, it shall bee DA-Aq =2DC+DB+Bq. Which rule is to be observed in every Transposition.

6. Thirdly, if the highest *species* of the Magnitude sought, be drawn into some Magnitude given; let there be made unto it, a *common Appliation* of all the Magnitudes of the Æquation. As BAq+BqA=Zc. It shall be $Aq+BA=\frac{Zc}{B}$.

7. Fourthly,

7. Fourthly, if it happen, that all the Magnitudes given be drawne into ſome degree of the Magnitudes ſought; let there be made a *Common Depreſſion* of them all, according to the order of the Table by applying them to the leaſt *ſpecies*. As Aqq+BAc=Zq Aq, ſhall be Aq+BA=Zq. putting out Aq in each of them. And after this manner any Æquation propoſed may be Depreſſed, or Reduced to leſſer *ſpecies*; If there be made a common Adplication, of all the termes, to the ſame degree. As Ac+XAq=Nc, divided by A, giveth $Aq+XA=\frac{Nc}{A}$. But divided by Aq, it makes $A+X=\frac{Nc}{Aq}$. Which Operation is of no ſmall uſe in the numerous Reſolution of Adfected Æquations: becauſe the Side ſought may be more eaſily eſtimated in leſſer *Poteſtates*, than in greater.

8. Fifthly, if any Magnitude be a Surde Side: the Æquation is to bee framed in the *Poteſtates* themſelves. √qBA+B=C. or by *Tranſpoſition*, √BA=C −B. And therefore their Quadrats, BA=Cq−2CB +Bq: or $A=\frac{Cq-2CB+Bq}{B}$.

Likewiſe √u: BA+CA:−D=B. or √u:BA+CA:=B+D. And therefore their Quadrats, BA+CA =Bq+2BD+Dq: or $A=\frac{Bq+2BD+Dq}{B+C}$.

Laſtly, $\sqrt{q\frac{A}{3}} = \sqrt{c\,2A}$. or by 11. C. XV. $\sqrt{qc\frac{Ac}{27}} = \sqrt{qc4Aq}$. Wherefore $Ac = 108\,Aq$: And $A = 108$.

9. The Conſtitution of Æquations, in which there are Three *ſpecies* aſcending in the order of the Scale, will appeare out of Sect. 2, 3, 4. Chap. XI. For becauſe.

$Z - A = E$: draw both parts into A.
$Z - E = A$: draw both parts into E.
$A - X = E$: draw both parts into A.
$E + X = A$: draw both parts into E.

And by this multiplication theſe Æquations will ariſe.

$ZA - Aq = Æ$	$Aq - XA = Æ$
$ZAq - Aqq = Æq$	$Aqq - XAq = Æq$
$ZAc - Acc = Æc$	$Acc - XAc = Æc$
&c	&c
$ZE - Eq = Æ$	$Eq + XE = Æ$
$ZEq - Eqq = Æq$	$Eqq + XEq = Æq$
$ZEc - Ecc = Æc$	$Ecc + XEc = Æc$
&c	&c

Whenſoever therefore an Æquation is propounded conſiſting of Three *ſpecies*, equally aſcending in the order of the Scale: You ſhall imagine the abſolute Magnitude given, to be a Rectangle under the two Magnitudes ſought; whether they be Sides, or Quadrats, or Cubes, &c. *viz.* ſuch as the *Poteſtas* of the middle *ſpecies*. But in the middle *ſpecies*, if the higheſt

higheſt *ſpecies* be negative, the coëfficient to bee the Summe of the Magnitudes ſought, and to be expounded of them both; But if the higheſt *ſpecies* be adfirmative, the coëfficient to be the difference of the Magnitudes ſought; and the *ſpecies* it ſelfe to be expounded of the greater, being negative; or of the leſſer, being adfirmative.

Then, the Summe and the Rectangle of two Magnitudes being given, their difference alſo is given: or the difference and the Rectangle being given, the Summe is given. For by 2 Chap. XI.

$\left.\begin{matrix} Q:\frac{1}{2}Z:-Æ=Q:\frac{1}{4}X: \\ Q:\frac{1}{2}X:+Æ=Q:\frac{1}{4}Z: \end{matrix}\right\}$ Where $\left.\begin{matrix} \sqrt{u:\frac{1}{2}Zq-Æ:}=\frac{1}{4}X. \\ \sqrt{u:\frac{1}{2}Xq+Æ:}=\frac{1}{4}Z. \end{matrix}\right\}$ foregoing.

Laſtly, the $\frac{1}{2}Z$ and $\frac{1}{4}X$ of two Magnitudes being given, the Magnitudes themſelves are given; by theſe two Rules.

I. Rule. $\frac{1}{2}Z \pm \sqrt{u:\frac{1}{2}Zq-Æ:}(\frac{1}{4}X) = \begin{matrix} A. \\ E. \end{matrix}$

II. Rule. $\sqrt{u:\frac{1}{2}Xq+Æ:}(\frac{1}{2}Z) \pm \frac{1}{4}X = \begin{matrix} A. \\ E. \end{matrix}$

10. GENESIS *Of the ſix Binomials out of their Surde Sides.*

The Rule is, $Z+2Æ=Zq$. But in Reſiduals $Z-2Æ=Xq$.

I. Quadrate the *Binomial*, $4+\sqrt{11}$. Here Z is $16+11$; that is, 27. And Æ is $\sqrt{16}\times\sqrt{11}$, that is, $\sqrt{176}$; whoſe double is $\sqrt{704}$. The Quadrat therefore ſhall be $27+\sqrt{704}$. Which is called I *Binomial*.

II. Ex-

II. Example. Quadrate the *former Bimedial* $\sqrt{qq12}+\sqrt{qq\frac{27}{4}}$. Here Z is $\sqrt{12}+\sqrt{\frac{27}{4}}$, or $\sqrt{\frac{48}{4}}+\sqrt{\frac{27}{4}}$; that is, $\sqrt{\frac{75}{4}}$. by 7. Chap. XV. And Æ is $\sqrt{qq12}\times\sqrt{qq\frac{27}{4}}$, or $\sqrt{qq3}\times\sqrt{qq27}$, that is, $\sqrt{qq81}$. *viz.* 3. the double whereof is 6. The Quadrat therefore shall be $\sqrt{\frac{147}{4}}+6$. Which is called II. *Binomial.*

III. Example. Quadrat *The Latter Bimedial*, $\sqrt{qq\frac{80}{3}}+\sqrt{qq15}$. Here Z is $\sqrt{\frac{80}{3}}+\sqrt{15}$ or $\sqrt{\frac{80}{3}}+\sqrt{\frac{45}{3}}$; that is, $\frac{\sqrt{245}}{3}$, by 7. Chap XV. And Æ is $\sqrt{qq\frac{80}{3}}\times\sqrt{qq15}$; or $\sqrt{qq80}\times\sqrt{qq5}$; that is, $\sqrt{qq400}$; to wit, $\sqrt{20}$. whose double is $\sqrt{80}$. The Quadrat therefore shall be $\sqrt{\frac{245}{3}}+\sqrt{80}$: which is called III *Binomial.*

In the Three others, which consist of the Roots of a *Binomial* and Residual joyned together, as $\sqrt{b:A+E:pl}\sqrt{r:A-E}$: it is cleere, that Z is 2A. And Æ is $\sqrt{:Aq-Eq}$. Wherefore.

IV. Exa. Quadrate *The Major*, $\sqrt{b:\frac{7}{3}+\sqrt{\frac{28}{9}}:pl:\sqrt{r:\frac{7}{3}-\sqrt{\frac{28}{9}}}}$: Here Z, is $\frac{7}{3}+\frac{7}{3}$, that is, 7. And Æ is: $\sqrt{\frac{49}{9}-\frac{28}{9}}$: that is $\sqrt{\frac{21}{9}}$, *viz.* $\sqrt{5}$; the double whereof is $\sqrt{20}$. The Quadrat therefore shall be $7+\sqrt{20}$. Which is called IV. *Binomial.*

V. Example. Quadrate *The Potent a Rational* with *a Medial*, $\sqrt{b:\sqrt{5}+1:pl\sqrt{r:\sqrt{5}-1}}$: Here Z is $\sqrt{5}+\sqrt{5}$; that is, $\sqrt{20}$. And Æ is $\sqrt{:5-1}$: that is, $\sqrt{4}$, *viz.* 2: The double wherof is 4. The Quadrat therefore shall be $\sqrt{20}+4$. Which is called V. *Binomial.*

VI. Example. Quadrate *The Potent two Medials*, $\sqrt{b:\sqrt{5}+\sqrt{3}:pl\sqrt{r:\sqrt{5}-\sqrt{3}}}$. Here Z is $\sqrt{5}+\sqrt{5}$, that is, $\sqrt{20}$. And Æ is $\sqrt{:5-3}$, that is, $\sqrt{2}$: The double

double whereof is $\sqrt{8}$, The Quadrat therefore ſhall be $\sqrt{20}+\sqrt{8}$: Which is called VI. *Binomial.*

11. ANALYSIS. In a Quadratic *Binomial* therefore, the greater *nomen* is Z.; and the leſſer *nomen* Æ. But in 2 Chap. XI. It is ordered, $\frac{1}{4}Zq-Æ=\frac{1}{4}Xq$: to wit, $\frac{1}{4}$ Q:A+E:--AE$=\frac{1}{4}$ Q:A-E: Wherefore if in ſtead of A and E, their Quadrats Aq and Eq, bee taken, it will be $\frac{1}{4}$Q:Aq+Eq: —AqEq$=\frac{1}{4}$Q:Aq-Eq: that is $\frac{1}{4}$ Z.q--Æq$=\frac{1}{2}$ X.q. Out of which Theoreme is deduced this Rule for the Analyſis of a *Binomial.*

$$\tfrac{1}{2}Z.\pm\sqrt{q:\tfrac{1}{4}Z.q-Æq:}\left(\tfrac{1}{2}X.\right)=\begin{cases}Aq.\\Eq.\end{cases}$$

I. Example. Seeke the Side of I *Binomial*, $27+\sqrt{704}$: namely Z.+2Æ. Wherefore $\frac{1}{2}$Z. is $\frac{27}{2}$: and Æ is $\sqrt{\frac{704}{4}}$: And $\frac{1}{4}$Z.--Æq is $\frac{729}{4}-\frac{704}{4}$; that is $\frac{25}{4}$; the Side whereof $\frac{5}{2}$ is $\frac{1}{2}$X.: But by the Rule, $\frac{27}{2}+\frac{5}{2}=\frac{16.\ 4.}{11.\ \sqrt{11.}}$ Wherefore the Side ſought is $4+\sqrt{11}$. And is called A *Binomial.*

II. Example. Seeke the Side of II. *Binomial*, $\sqrt{\frac{147}{4}}+6$, namely Z.+2Æ. Wherefore $\frac{1}{2}$Z. is $\sqrt{\frac{147}{16}}$: And Æ is 3: And $\frac{1}{2}$Z.q--Æq is $\frac{147}{16}-(9)\frac{144}{16}$; that it $\frac{3}{16}$: the Side whereof $\sqrt{\frac{3}{16}}$ is $\frac{1}{2}$X. But by the Rule:

$$\sqrt{\tfrac{147}{16}}+\sqrt{\tfrac{3}{16}}=\begin{matrix}\sqrt{12.}\\\sqrt{\frac{27}{8}.}\end{matrix}\begin{matrix}\sqrt{qq\ 12}\\\sqrt{qq\ \frac{27}{4}}\end{matrix}$$

Wherefore the Side ſought is $\sqrt{qq\ 12}+\sqrt{qq\ \frac{27}{4}}$, and is called the *Former Bimedial.*

III. Example. Seeke the Side of III *Binomial.* $\sqrt{\frac{45}{2}}+\sqrt{80}$, namely Z.+2Æ. Wherefore $\frac{1}{2}$Z. is $\sqrt{\frac{45}{16}}$

And

And Æ is $\sqrt{20}$. And $\frac{1}{4}$ Zq—Æq is $\frac{245}{12}$—(20) $\frac{240}{12}$, that is, $\frac{5}{12}$ the Side whereof $\sqrt{\frac{5}{12}}$ is $\frac{1}{2}$ X. But by the Rule: $\sqrt{\frac{24}{12}} \pm \sqrt{\frac{5}{12}} = \left.\begin{matrix}\sqrt{\frac{80}{3}}.\ \sqrt{qq\frac{80}{3}} \\ \sqrt{15}.\ \sqrt{qq15}\end{matrix}\right\}$ The Side sought therefore is $\sqrt{qq\frac{80}{3}} + \sqrt{qq\ 15}$. and is called the *Latter Bimedial.*

IV. Example. Seek the Side of IV *Binomial*, $7+\sqrt{20}$, namely Z+2Æ. Wherefore $\frac{1}{2}$ Z is $\frac{7}{2}$: And Æ is $\sqrt{5}$: And $\frac{1}{4}$ Zq—Æq is $\frac{49}{4}$—(5) $\frac{20}{4}$; that is $\frac{29}{4}$: the Side whereof $\sqrt{\frac{29}{4}}$ is $\frac{1}{2}$X. But by the Rule: $\frac{7}{2} \pm \sqrt{\frac{29}{4}} = \left.\begin{matrix}\frac{7}{2}+\sqrt{\frac{29}{4}}.\ \sqrt{b}:\frac{7}{2}+\sqrt{\frac{29}{4}} \\ \frac{7}{2}-\sqrt{\frac{29}{4}}.\ \sqrt{r}:\frac{7}{2}-\sqrt{\frac{29}{4}}\end{matrix}\right\}$ The Side sought therefore is $\sqrt{b}:\frac{7}{2}+\sqrt{\frac{29}{4}}$: pl $\sqrt{r}:\frac{7}{2}-\sqrt{\frac{29}{4}}$. And is called the *Major.*

V. Example. Seeke the Side of V *Binomial*, $\sqrt{20}+4$, namely Z+2Æ. Wherefore $\frac{1}{2}$ Z is $\sqrt{5}$: And Æ is 2. And $\frac{1}{4}$Zq—Æq is 5-4; that is, 1: the Side whereof 1 is $\frac{1}{2}$X. But by the Rule: $\sqrt{5} \pm 1 = \left.\begin{matrix}\sqrt{5}+1.\ \sqrt{b}:\sqrt{5}+1: \\ \sqrt{5}-1.\ \sqrt{r}:\sqrt{5}-1:\end{matrix}\right\}$ The Side sought therefore $\sqrt{b}:\sqrt{5}+1$: pl $\sqrt{r}:\sqrt{5}-1$. And is called *The Potent a Rationall* with *a Medial.*

VI. Example. Seeke the Side of VI *Binomial*, $\sqrt{20}+\sqrt{8}$, namely Z + 2Æ. Wherefore $\frac{1}{2}$ Z is $\sqrt{5}$: And Æ is $\sqrt{2}$. And $\frac{1}{4}$ Zq-Æq is 5—2; that is 3: whose Side $\sqrt{3}$ is $\frac{1}{2}$X. But by the Rule: $\sqrt{5} \pm \sqrt{3}$. $= \left.\begin{matrix}\sqrt{5}+\sqrt{3}.\ \sqrt{b}:\sqrt{5}+\sqrt{3} \\ \sqrt{5}-\sqrt{3}.\ \sqrt{r}:\sqrt{5}-\sqrt{3}\end{matrix}\right\}$ The Side sought therefore is $\sqrt{b}:\sqrt{5}+\sqrt{3}$: pl $\sqrt{r}:\sqrt{5}-\sqrt{3}$. And is called *the Potent two Medials.*

12. And here by the way the *Genesis* of a plaine Rect-

Rectangle Triangle doth offer it ſelfe to bee conſidered.

Becauſe Zq=Xq+4Æ : namely Hq=Bq+Cq, by47.e.1. There being propoſed any two lines or numbers whatſoever, A and E; the Sides of a Rectangle Triangle ſhall be A+E, A−E, √4AE : or (changing A and E into Aq & Eq) Aq+Eq, Aq−Eq, 2AE. As if the two numbers propoſed be 2 and 1; the Sides ſhall be 3, 1, √8 (namely 2+1, 2−1, √:4×2×1) or 5, 3, 4, (namely 4+1, 4−1, 2×2×1.

13. Two Rectangle Triangles being given, H, B, C; h, b, ¢ : to frame a third of them: and that two wayes. Firſt, becauſe Bq=Hq−Cq } Multiply them one into another; and there ſhall be.

And bq=hq−¢q }

Bqbq=Hqhq+Cq¢q mi Hq¢q+Cqhq.

But Hqhq+Cq¢q+2HCh¢=Q:Hh+C¢: }

And H¢qq+Cqhq+2HCh¢=Q:H¢+Ch: }

Subduct one Quadrat out of the other; and there ſhall be

Bqbq=Q:Hh+C¢:mi Q:H¢+Ch.

And ſo out of theſe, is found a third Triangle,

Bb.Hh+C¢.H¢+Ch. Let this be Rule I.

And it is expreſſed in words thus: For the Baſe of the new Triangle, *Take* the Rectangle under the Baſes: For the Hypotenuſa, a Rectangle under the Hypotenuſes, increaſed with a Rectangle under the Cathetus: For the Cathetus, a Rectangle under the Hypotenuſa of the Firſt and Cathetus of the Second, in-

increaſed with a Rectangle under the Cathetus of the Firſt, and Hypotenuſe of the Second.

II. Becauſe Hq = Bq + Cq } Multiply them one into another, and there
And hq = bq + cq } ſhall be.

Hqhq = Bqbq + Cqcq pl Bqcq + Cqbq.
But Bqbq + Cqcq - 2BCbc = Q:Bb - Cc: }
And Bqcq + Cqbq + 2BCbc = Q:Bc + Cb: }
Adde theſe two Quadrats; and there ſhall be
Aqhq = Q:Bb — Cc: pl Q:Bc + Cb:
And ſo out of theſe is found a third Triangle,
Hh. Bb - Cc. Bc + Cb. Let this be Rule II.

And it is expreſſed in words thus; For the Hypotenuſes of the new Triangle, *Take* a Rectangle under the Hypotenuſe. For the Baſe, a Rectangle under the Baſes, leſſened by a Rectangle under the Cathetus. For the Cathetus, a Rectangle under the Baſe of the Firſt, and Cathetus of the Second, increaſed with a Rectangle under the Cathetus of the Firſt and Baſe of the Second.

14. If the Sides of a Rectangle Triangle be continually multiplied according to the two Rules now found out; the Firſt multiplication ſhal Double the angle oppoſite to the Baſe; the Second ſhall Triple it; the Third ſhall Quadruple it; and ſo ſtill farther.

Exampe

Examp. Rule. I. Bb.Hh+Cc.Hc+Ch.

B. H. C: Of a Single angle.
B. H. C.

Bq. Hq+Cq. 2HC: Double angle
B. H. C.

Bc. Hc+HCq. 2HqC
+2HCq. HqC+Cc.

Bc. Hc+3HCq. 3HqC+Cc. Triple angle.
B. H . C.

Bqq. Hqq+3HqCq. 3HcC+HCc.
Cqq+3HqCq. HcC+3HCc.

Bqq. Hqq+6HqCq+Cqq.4HcC+4HC. Quadr. angle
B. H. C. &c.

Examp. Rule II. Hh. Bb-Cc.Bc+Cb.

H. B. C. Of a Single angle.
H. B. C.

Hq. Bq-Cq. 2BC. Double angle.
H. B. C

Hc. Bc-BCq. 2BqC
-2BCq. BqC-Cc.

Hc. Bc-3BCq. 3BqC-Cc. Triple angle.
H. B C

Hqq. Bqq-3BqCq. 3BcC-BCc.
+Cqq-3BqCq BcC-3BCc.

Hqq. Bqq-6BqCq+Cqq.4BcC-4BCc: Quadr. angle.
H. B C &c.

CHAP.

CHAP. XVII.

Another view of the latter Table in Chap. XII. *concerning Æquations.*

1. FRom a Binomial Root A+E, all the *ſpecies* of *Poteſtates* are affirmative. And from a Reſiduall Root, all the *ſpecies* of *Poteſtates* are alternely negative. As Q:A−E: is Aq−2AE+Eq. And C:A−E: is Ac−3AqE+3AEq−Ec. And QQ:A−E: is Aqq −4AcE+6AqEq−4AEc+Eqq. &c. So that if the *ſpecies* of any *Poteſtas*, taken alternely, bee gathered into two Summes: theſe Summes joyned together with the ſign of the Root, ſhall be the *Poteſtas* of the Root. And this is the conſtitution of Quadratic, Cubic, and other Binomials and Reſiduals.

2. Wherefore the difference of the *Nomina*, of every Binomiall or Reſiduall, is an Homogeneal *Poteſtas* of the difference of the *Nomina* of the Root. *viz:* Ac+3AEq mi 3AqE+Ec, or Ac+3AEq−3AqE−Ec, is C:A−E.

3. And the difference of the Quadrats of the *Nomina* of every Binomiall or Reſiduall, is an Homogeneal *Poteſtas* of the difference of the Quadrats of the *Nomina* of the Root: To wit, Q: Ac+3AEq: mi Q: 3AqE+Ec; is C:Aq−Eq. For by Examp: Rule I, in 14 Chap: XVI, if A be conceived to bee the Hypotenuſe of a rectangled triangle, and E the Cathetus and

Aq

Aq+Eq the Quadrat of the Baſe: this Theoreme ſhall be otherwiſe expreſſed by Symbols: Thus,
Q: Hc+3HCq: mi Q:3 HqC+Cc = C:Bq = Q: Bc,
Therefore.

4. But if the *ſpecies* aggregated in the *Nomina*, be themſelves alſo alternely affirmed and denyed: the Summe of the Quadrats of the *Nomina*, is an Homogeneal *Poteſtas* of the Summe of the Quadrats of the *Nomina's* of the Root. *viz*: Q: Ac-3AEq: pl Q: 3AqE-Ec: is C:Aq-Eq. For by Examp: Rule II, in 4. Chap. XVI. If A conceived to bee the Baſe of a Rectangled triangle; and E the Cathetus; and Aq+Eq the Quadrat of the Hypotenuſe: this Theoreme ſhall be otherwiſe expreſſed by Symbols: Thus,
Q: Bc-3BCq: pl Q:3BqC-Cc: = C: Hq = Q: Hc:
Therefore theſe are excellent myſteries touching angular Sections.

5. All the Intermediall or middle *ſpecies* of each order, are alſo *Poteſtates* of Meane Proportionalls betweene A and E. *viz*: between Ac and Ec there are two Meane Proportionals; AqE and AEq: which alſo are Cubes of M and N. Wherefore A, √cAqE, √cAEq, E, are continually Proportionals, namely A, M, N, E. for AqE = AMN = Mc. & AEq = MNE = Nc.

And from hence appeareth the invention of how-many-ſoever Meane Proportionals between A and E. As, if you deſire five meane-Proportionals, the *Poteſtates* ſhall be $\sqrt[6]{}$ or ec: whoſe Index exceedeth, by a unit, the number of Meane-Proportionals deſired.

And there shall be A, √ccAqcE, √ccAqqEq, √cc,AcEc,√ccAqEqq,√cc,AEqc, E, &c.

6. Each Middle *species* of every *Kinde* is made up of two *Potestates* of the *Nomina* of the Root, whose Indices put together, are equall to the Index of the same *kinde*: but the Distances of the same Middle *species* from his Extremes, are equall to the Indices of each others *Factores*: and doe answer to their owne *Factores* in a common Angle. *Scil.* AqEc of the Quadrato-cubic *kinde*, is made up of Aq into Ec, to both which it answereth in a common angle.

And is the third from Aqc: and the second from Eqc.	Aq	Ac	Acq	Aqc 5√ÆAcc
CONSECTARIE.	A	.	.	10ÆAc
And from hence it will be easie to frame any *Potestas* of a Binomiall Root given.	√Æ	Æ	.	10√ÆcAqq
		√Æc	Æq	5ÆqA √Æqc

As in Example, If the Quadrato-Cube of the Binomial A+√Æ, be sought, it shall be Aqc+10ÆAc+5ÆqA pl: 5√ÆAqcc+10√Æc Aqq+√Æqc, which is a Binomial Quadrato-cubicated.

7. If any *species* be multiplied by Æ, the Magnitude produced shall be a midle Collaterall *species*, in the next row save one, and the same in number from its Extremes: Ac. Æ is AqqE, which is the first from Aqc and the fourth from Eqc. So AcE. Æ is AqqEq which is the second from Acc; and the fourth from Ecc. And in like manner of the rest.

8. I

8. If any *species* be multiplied by A-E, or X: the Magnitude produced ſhall be the Difference between two *ſpecies* of the row following next it, on both Sides. As AcX=Aqq-AcE. AqEX=AcE-AqEq. AEqX=AqEq-AEcEcX=AEc-Eqq.

Wherefore if all the *ſpecies* of any order be multiplied by X, there will be produced the difference of the two extreme *Poteſtates* of the row next above it. Ac+AqE+AEq+Ec being drawne into X, make Aqq-Eqq.

9. In the rowes which have their Indices odde, (1, c, qc, &c.) the Summe of the two Extreme *Poteſtates*; but in the orders of even Indices (q, qq, cc, & c.) the difference of two Extreme *Poteſtates*; is made of A+E drawne into every one of the *ſpecies* of the precedent row, alternly adfirmed and denied: As of Aq-AE+Eq drawne into A+E, ſhall be made Ac+Ec And of Ac-AqE+AEq-Ec, drawne into A+E, ſhall be made Aqq-Eqq.

10. If the ſame magnitude be multiplied by two contrary magnitudes: the magnitudes made of them ſhall alſo be contrary. As Aq-2AE+Eq, drawn into A-E, ſhall make Ac-3AqE+3AEq-Ec. But the ſame drawn into A+E, ſhall make Ac+3AqE-3AEq+Ec.

11. *Unio*, or Numbers ſet before the *Species* are Numerarie Figures. For all under A and E are Roots: all under Aq and Eq, are Triangulars; all under Ac and Ec, are Pyramidals; all under Aqq and Eqq are

Triangulo-triangulars: all under Aqc and Eqc are Triangulo-pyramidals: all under Aoc and Ecc, are Pyramidi-pyramidalls, &c.

12. If a Root consist of three *Nomina* A, E, I.

Quadr. { Aq, 2AE / Eq, 2EI / Iq, 2AI }

Quad. { Ac, Ec, Ic / 3AqE, 3AEq / 3AqI, 3AIq / 3EqI, 3EIq / 6AEI. }

And note, that if in any *Species*, the number of the negative Sides be odde; that *Species* shall be negative: As Q:A+E—I: is Aq+2AE+Eq—2EI+Iq—2AI. And C.A+E—I = Ac+3AqE+3AEq+Ec—3EqI: +3EIq—Ic-3AqI+3AIq—6AEI.

Chap. XVIII.

THE ANALYTICAL STORE.

1. OUt of the first and most easie *Æquations*, which are nothing else, but either expositions of the termes, or simple effections (such are $\frac{1}{1}Z$ —E=$\frac{1}{1}X$: and $\frac{1}{1}X+E=\frac{1}{1}Z$: and those of Chap: XI; and such like) innumerable others are deduced by Addition, Subduction, Multiplication, Division, Transposition, and Interpretation: taking that which is found equall to another, instead of that, to which it is equall. Which Analyticall furniture is no lesse precious then plentious: I shall record some of the chiefest and most necessary peeces thereof: the studious

dious in Analytice will deviſe more for their exerciſe. And whereſoever, whether in Arithmetique, Geometrie, or in any other Art, he ſhall light upon ſome magnitude, to which another is underſtood to bee equall; he ſhall turne, wreſt and vary, that Æquality by whatſoever meanes and comparings he can; that from thence he may find out a new Inſtrument of Art; which afterwards he ſhall keep in Store; and whereſoever occaſion ſerves, bring it forth for the helpe and advancement of the Art.

2. $Q:1 = 9Q:\frac{1}{3}$: &c. $C:1 = 27C:\frac{1}{3}$: &c.

$Q:1 = \frac{1}{3}Q:3$: &c. $C:1 = \frac{1}{27}C:3$: &c.

$Q:1 = \frac{1}{4}Q:\frac{1}{4}$: &c. $C:1 = \frac{1}{64}C:\frac{1}{4}$: &c.

$\frac{1}{4}Q:1 = \frac{1}{4}*\frac{1}{4}Q:\frac{1}{4}$: &c. $\frac{1}{4}C:1 = \frac{1}{4}*\frac{1}{64}C:\frac{1}{4}$: &c.

$\frac{1}{3}Q:4 = \frac{2*16}{3}Q:1$: &c. $\frac{1}{3}C:4 = \frac{2*64}{3}C:1$: &c.

3. If a Right line be cut into equall & unequal parts; the Rectangle under the unequal ſegments, is equall to the Difference of the Quadrats of the Biſegment, and the Interſegment: that is, to the Quadrate of half the Summe, and half the Difference of the Segments. 5e2.

$AE = Q:\frac{1}{2}A+\frac{1}{2}E$: mi $Q:\frac{1}{2}A-\frac{1}{2}E$: that is, $AB = \frac{1}{4}Zq-\frac{1}{4}Xq$.

4. If a Line cut in Halfes, be increaſed; the Rectangle under the whole increaſed, and the increaſe, is equall to the Difference of the Quadrats of the halfe increaſed, and of the halfe. 6e2.

$A+E$ into $E = Q:\frac{1}{2}A+E$: mi $Q:\frac{1}{2}A$: And $A+E$ into $A = Q:\frac{1}{2}E+A$: mi $Q:\frac{1}{2}E$.

There being given therefore the Summe of Three

-:: (Aq+AE+Eq) with either of the Extremes; the Two others are also given. For

√u:Aq+Æ+Eq-¼Aq:mi¦A=E.

√u:Aq+Æ+Eq-¼Eq:mi¦E=A.

5. If a Line be cut howsoever, the Summe of the Quadrats of the whole, and one of the Segments, is equall to the Aggregate of two Rectangles under the whole Line, and the same Segment, and with the Quadrate of the other Segment. 7 e 2.

Zq+Aq=2ZA+Eq. And Zq+Eq=2ZE+Aq.

Wherefore 2ZA+Eq+Aq=Zq=2ZE+Aq-Eq.

6. If a Line cut howsoever, be increased with one of his Segments; foure Rectangles contained under the Line cut, and the Segment increasing, are equall to the Difference of the Quadrats of the whole increased, and the other Segment. 8 e 2.

Q:Z+E:-Aq=4ZE. and Q:Z+A:-Eq=4ZA.

7. If a line be cut, both equally and also unequally, the Summe of the Quadrats of the unequall Segments, is equall to Double the Summe of the Quadrats of the Bisegment, and of the Intersegment. 9 e 2.

Aq+Eq=2Q:½A+½E:+2Q:½A-½E.

8. If a Line cut in Halfes be increased; the Summe of the Quadrats of the whole increased, and of the Increase, is equall to Double the Summe of the Quadrats of the Halfe increased, and of the Halfe. 10 e 2.

Q:A+E:+Eq=2Q:½A+E:+2Q:½A:

Q:A+E:+Aq=2Q:½E+A:+2Q:½E.

9. Aq-ZA-AE=XA+AE=¼ZA+¼XA=Q:Z-E:=Q:E+X:=Z-Eq=Eq+X. And

And $Eq = ZE - AE = AE - XE = \frac{1}{2}ZE - \frac{1}{2}XE = Q:Z-A = Q:A-X: = Z - Aq = Aq - X.$

10. $Æ = \frac{1}{2}Zq - \frac{1}{2}Xq = ZA - Aq = ZE - Eq = Aq - XA = Eq + XE = \frac{1}{2}Zq - \frac{1}{2}Z = \frac{1}{2}Z - \frac{1}{2}Xq = \frac{1}{2}ZA - \frac{1}{2}XA = \frac{1}{2}ZE + \frac{1}{2}XE.$

11. $Z = Aq + Eq = Zq - 2AE = 2AE + Xq = ZE + XA = ZA - XE = 2Q:\frac{1}{2}Z: + 2Q:\frac{1}{2}Z - E: = Q:A - 2N: + Q:2M - E: = \frac{1}{2}Zq + \frac{1}{2}Xq = 2Q:\frac{1}{2}Z: + 2Q:\frac{1}{2}X.$ Out of theſe two laſt Æquations draw this Conſectarie.

If a Magnitude conſiſt of the Quadrats of two Magnitudes, the Double thereof ſhall alſo conſiſt of two Quadrats, namely, of the Summe and of the Difference. And the halfe thereof ſhall alſo conſiſt of two Quadrats, namely, of Halfe the Summe, and Halfe the Semi-difference.

And $X = Aq - Eq = ZX = 2XA - Xq = 2XE + Xq = ZA - ZE = XA + XE = Zq - 2ZE = ZA + XE - 2Æ = XA + 2Æ - ZE = Q:A + 2N:$ mi $Q:2M + E.$

12. $Q:\frac{1}{2}A + \frac{1}{2}E: = Q:\frac{1}{2}A - \frac{1}{2}E: + Æ.$
And $Q:\frac{1}{2}A - \frac{1}{2}E: = Q:\frac{1}{2}A + \frac{1}{2}E: - Æ.$

13. $2A+2E$ into $A = 2Aq + 2AE = Zq + X.$
And $2A-2E$ into $A = 2Aq - 2AE = X + Xq.$
And $2A+2E$ into $E = 2AE + 2Eq = Zq - X.$
And $2A-2E$ into $E = 2AE - 2Eq = X - Xq.$

14. $Xq = ZqXq = Z + 2Æ$ into $Z - 2Æ = Zq - 4Æq.$

15. $ZÆ = AqE + AEq.$ And $XÆ = AqE - AEq.$
And $ZÆ = AcE + AEc.$ And $XÆ = AcE - AEc.$
Wherefore $Z + 3ZÆ = Zc.$ And $Z - 3XÆ = Xc.$

And $ZZ = z^{+}ZÆ = Ac^{+}AqE^{+}AEq+Ec.$
And $ZX = x - XÆ = Ac - AqE + AEq - Ec.$
And $XZ = x + XÆ = Ac + AqE - AEq - Ec.$
And $XX = z - ZÆ = Ac - AqE - AEq + Ec.$
Hence $ZZ + XX = 2z.$ And $XZ + ZX = 2x.$
And $ZZ - XX = 2ZÆ.$ And $XZ - ZX = 2XÆ.$

16. If in a Circle 7.22::$\delta.\pi$::113.355. it shall bee $\delta.\pi$:: 2R.P: And

$\delta.\pi$::Rq.	Circl.	And $\pi.\delta$::$\frac{1}{4}$Pq.	Circl.
$\delta.\pi$::2Rc.	Cylind.	And $\pi q.\delta q$::$\frac{1}{2}$Pc.	Cylind.
$\delta.\pi$::$\frac{4}{3}$Rc.	Sphær.	And $\pi q.\delta q$::$\frac{1}{4}$Pc.	Sphær.
$\delta.\pi$::$\frac{1}{3}$Rc.	Con.	And $\pi q.\delta q$::$\frac{1}{12}$Pc.	Con.

17. Moreover it is necessary, for him, that intends to be an Analyst, not to be ignorant of these Geometricall Theoremes and Problemes.

1. Theor: Triangles are equall; If in both, either the three Sides; or two Sides, with the angle comprehended; or two Sides, with the angle opposed to the same Side; or two angles, with the interjacent Side; or two angles, with the Side subtended to the same angle; be equall. 4,8,26, e 1.

2. Theor: Plaine Triangles are like: If either they be Equiangled; or have all their Sides Proportional; or have one angle equall, and another angle of Proportionall legges, and the third angle Homogeneal. 4,5,6,7, e 6.

3. Theor: In every Triangle; the greater Side is subtended to the greater Angle; and the Lesse to the Lesser; and the Equall to the Equall. 18,19, e 1.

4. Theor.

4. Theor: Two right lines are parallell; If a right line cutting them, make equall, either the alterne angles; or the externe and the interne oppoſite; or the two interne, on the ſame Side, to two right Angles; And contrariwiſe. 27,28,29,30, e 1. For lines parallell are as it were one broad line.

5. Theor: The three Angles of a Triangle, put together, are equall to two right Angles: and (one Side being produced) the Externe angle is equall to the two Interne oppoſite. 32. e 1.

6. Theor. If a Right line out of the Center, cut into Halfes a Right line inſcribed in a Circle: it ſhall cut it at Right angles. 3 e 3.

7. Theor: A Perpendicular upon the end of the Diameter, toucheth the Circle. 16,18,19, e 3.

8. Theor: The Angle at the Center is double the angle at the Circumference. 20 e 3.

9. Theor: In the ſame or equall circles, angles upon equall arches, are equall. 21 e 3.

10. Theor: An angle in a Semicircle, is a Right angle. 31 e 3.

11. Theor: If from a Point in the Circumference of a Circle, there be drawne two right lines, the one touching the Circle, and the other cutting it: the meaſure of the angle comprehended between them, ſhall bee equall to halfe the Circumference cut off. Inſtead of 32 e 3.

12. Theor. Triangles, or Parallelogrammes, of equall height, or between the ſame parallels, are to one another,

another as their Bases. 35,36,37,38, e 1. and 1 e 6.

13. Theor: A right line cutting an angle of a Triangle in halfe, cutteth the Base proportionally to the Legges. 3 e 6.

14. Theor: Every rectangle Triangle may be noted with the letters ABC: so that A be the Right angle; and BA the Base: and CA the Cathetus, and BC the Hypotenuse.

15. Theor: In a plaine rectangled Triangle, a Perpendicular let fall from the Right angle upon the Hypotenuse, divides the triangle into two Triangles, both like to the whole, and to one another, 8 e 6.

BC. BA. CA:: BA. BP. AP:: CA. AP. CP.

Hypotenuse. Bases. Catheti.

From hence it follows, that

1. The Perpendicular is a meane-proportionall between the Segments of the Hypotenuse. And therefore the Quadrate of the Perpendicular is equall to the Rectangle under the Segments. Scil: ∴ BP, AP, CP. APq = BP × CP.

2. The Base is a Meane-proportionall between the Hypotenuse, and the Segment of the Hypotenuse on the Bases Side. Scil: ∴ BC, BA, BP.

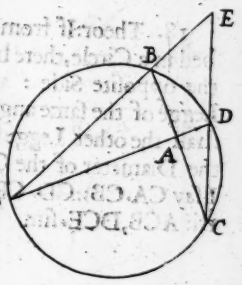
3. The Cathetus is a Meane-proportionall betweene the Hypotenuse, and the Segment of the Hypotenuse on the Cathetus Side. Scil: ∴ BC, CA, CP.

4. The Quadrats of the Base and Cathetus, are as the

the Segments of the Hypotenuſe on their reſpective Sides. BP.CP::BAq.CAq.

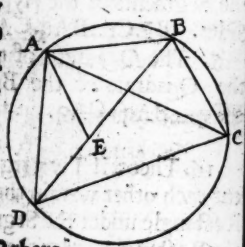
5. The Quadrate of the Hypotenuſe is equall to the Quadrats of the Baſe and Cathetus together. BCq=BAq+CAq.

16. Theor: If Two right lines inſcribed in a circle, cut each other within the Circle (in the point A); the Rectangle under the Segments of the one, is equall to the Rectangle under the Segments of the other. 35e3. But if they cut one another without the Circle (in the point E) the Rectangles under the Segments of them both from the point to the Convexe and Concave of the Circle, are equall: 36, 37e3. I ſay firſt AB*AC =AD*AF. for tri: BAF, DAC ſim. I ſay 2ly. EB*EF =ED*EC. for tri: BEC, DEF. ſim.



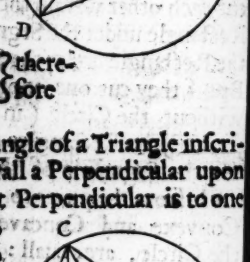
17. Theor: In a Quadrangle inſcribed in a Circle, the Interior oppoſite angles together, are equall to two right angles. 22e3. And if there bee drawne Two Diagonall lines, the Rectangle under the Diagonalls, is equall to the two Rectangles under the

the oppoſite Sides. I ſay AC×BD = AB×CD + AD×BC. For tri: ACB, ADE ſim. and tri: ADC, AEB ſim.



Where-fore { AC.CB::AD.DE } there-fore
{ AC.CD::AB.BE }

18. Theor: If from any angle of a Triangle inſcribed in a Circle, there be let fall a Perpendicular upon the oppoſite Side: as that Perpendicular is to one Legge of the ſame angle; ſo ſhall the other Legge bee to the Diameter of the Circle. I ſay CA.CB::CD.CE. For tri: ACB, DCE. ſim.

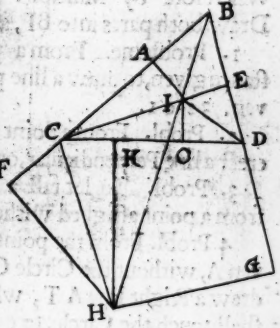


19. Theor. Triangles having one equall angle, have that Ration to one another, which is compounded of the Sides. 23 e 6.

20. Theor. If Halfe the Summe of the Three Sides of a Plaine triangle, and the Three Differences of the Three

Three Sides from that halfe Summe, be continually drawne into one another ; Or thus, if, taking any Side of a Triangle for the Baſe, and the two others for the Legges; A Rectangle under the halfe Summe and halfe Difference of the Summe of the Legs and Baſe, be drawne into a Rectangle under the halfe Summe and halfe Difference of the Baſe, and of the Differences of the Legges: The Quadrat Side of the Product ſhall be equall to the *area* (or content) of the Triangle.

Let BCD be a Triangle, whereof the Legges are BC and BD, and CD the Baſe. Cut the Three angles into halfes with the right lines BI, CI, DI, concurring in I: from whence to the Sides, at Right angles, draw the lines IA, IE, IO. There are therefore within the Triangle BCD, Three paire of equall Triangles.



Wherefore, if to the Legges BC be joyned aright, CF = DE; it ſhall be BF = ½BC + ½BD + ½CD:
And BA = BF - CD = ½BC + ½BD - ½CD:
And AC = BF - BD = ½CD + ½BC - ½BD:
And CF = BF - BG = ½CD - ½BC + ½BD:

Mea-

Measuring BG=BF: And CK=CF. Draw Perpendiculars FH, GH, KH: And Protract BI unto H. Becauſe the angles FCK+FHK=2 right angles =FCK+ACO: And ang: ACO+AIO=2 right; the quadrangles FCKH, AIOC ſhall be ſim: and tri: CFH, IAC ſim. the tri: BAI, BFH are alſo ſim.

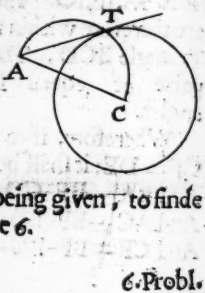
Theſe declared. I ſay, That the Quadrat of the *area* of the triangle, namely BFq×IAq=BF×BA×AC×CF. For IA. BA::FH.BF } becauſe tri. ſim.
And IA.AC::CF.FH }
Wherefore by Multipl: IAq×BF=BA×AC×CF. Draw both parts into BF, and there ſhall be &c.

1. Probleme. From a point given, or at a Diſtance given, to draw a line parallel to a Right line given. 32. e 1.

2. Probl. From a point given in a Right line, to erect a line Perpendicular, or at Right angles. 11. e 1.

3. Probl. To let fall a Perpendicular line given, from a point aſſigned without it. 12 e 1.

4. Probl. From the point given A, without the Circle C, to draw a Right line A T, which ſhall touch the Circle. 17 e 3.

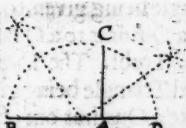


5. Probl. Three right lines being given, to finde a Fourth Proportionall line. 12 e 6.

6. Probl.

6. Probl. Two right lines being given AB, AD, to find out a Meane-Proportionall line. 13 e 6.

7. Probl. Two right lines AB, AC, or AD, AC, being given, to finde a Third line continually Proportionall. 11 e 6.

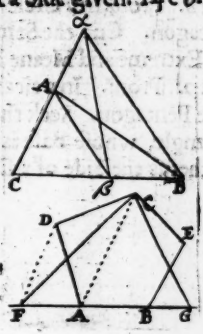


8. Probl. A Triangle being given, whose height is AC, and halfe the Base AB, to frame a Quadrat ADq equall thereto.

9. Probl. A Rectangle being given, to frame another Rectangle equall to it, at a Side given. 14 e 6.

10. Probl. A triangle being given, to frame another Triangle equall to it, at a height given.

From the points of height A and α, let the lines Aβ, and αB drawne to the opposite angles, be parallel.



11. Probl. A Polygon being given, to frame a Triangle equall to it.

12. Prob.

12. Probl. Three points being given, not lying direct, to draw a circumference through them. 25 e 3.

13. Probl. The Baſe and Cathetus of a rectangled Triangle being given, to finde the Hypotenuſe: or to adde a Quadrat to a Quadrat.

14. Probl. The Hypotenuſe and Baſe of a rectangled Triangle being given, to finde the Cathetus; or to take a Quadrat out of a Quadrat.

15. Probl. To finde the Ration of Two like Figures; ſeeke a Third Proportional.

16. Probl. A Figure being given, to frame a like Figure, in a Ration given. Seeke a Meane-proportional betweene the Side thereof, and a like Side.

17. Probl. In a circle given to inſcribe an ordinate or regular Hexagon. 15 e 4.

18. Probl. In a circle given to inſcribe an ordinate Decagon. Cut the Semidiamiter of the Circle after the Extreme and Meane Ration, by 11 e 2.

19. Probl. In a circle given to inſcribe an ordinate Pentagon. Seek the Hypotenuſe of a rectangled Triangle, whoſe Baſe is the Side of an Hexagon, and Cathetus the Side of a Decagon.

Chap. XIX.

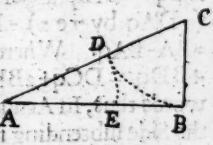
Examples of Analytical Æquation, for inventing of Theoremes, and reſolving of Problemes. At which marke (as it were) the Precepts hitherto delivered, do principally aime.

Probl. I. THE Invention of 11 e 2. Namely, to cut B a Right line given, ſo that the Rectangle under the whole B, and the leſſer Segment, may be equall to the Quadrat of the greater Segment.

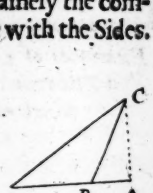
Let the greater Segment be put A: the leſſer ſhall be B—A. Draw B—A into B; and there ſhall bee made Bq—BA=Aq: or Aq+BA=Bq. Wherefore √u:Bq+¼Bq:-½B=A; by 9 Ch. XVI.

Which Theoreme is expreſſed by words thus: If to the Quadrat of the Line given, be added a quarter of the ſaid Quadrat; and from the Quadrat-ſide of the Summe, be taken Half the line given; the Remainder ſhall be the greater Segment.

Now it is Geometrically effected thus; Draw AB=B: And to it at Right angles, ſet BC=½B; and draw the Hypotenuſe AC: It ſhall be AC=√u:Bq+¼Bq. Cut off CD=BC; And the Remainder ſhal be AD=√u:Bq+¼Bq:-½B. Laſtly, meaſure AE=AD, for the greater Segment.

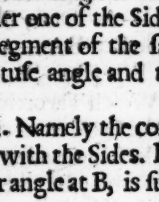


Probl. II. Invention of 12 e 2. Namely the comparing of the Base of an obtuse Angle with the Sides. Let BCD be a Triangle, whose interior Angle at B is obtuse; the Base is DC, and the Sides BD, BC. Here BCq—BAq=CAq=DCq (-DAq by 4 e 2)-BDq-2BD*BA -BAq. Wherefore BCq+BDq =DCq-2BD*BA.



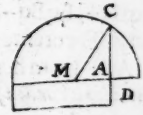
Which Theoreme is expressed by words, thus; In Obtusangled Triangles, the Quadrat of the Side subtending the obtuse angle, exceedeth the Summe of the Quadrats of the Sides comprehending the same angle, by a Double Rectangle under one of the Sides about the obtuse angle, and the Segment of the said Side (produced) betweene the obtuse angle and the Perpendicular.

Probl. III. Invention of 13 e 2. Namely the comparing the Base of an acute Angle with the Sides. Let BCD be a Triangle, whose interior angle at B, is supposed, sharpe. The Base is DC, and the Sides BC, BD. Here BCq —BAq = CAq = DCq (-DAq by 7 e 2) - BDq+2BD *BA-BAq. Wherefore BCq +BDq=DCq+2BD*BA. In words thus, In Acutangled Triangles, the Quadrat of the Side subtending the Acute angle, is lesse than the Summe of the Quadrats, &c.



Probl.

Probl. IV. Invention of 14 e 2. Namely a Quadrat equall to the Rectangle AB × AD. Bee it AB+AD=2BM. Wherefore AB+AD is cut into equall parts at M, and unequall at A. It shall be therefore (by 5 e 2) AB × AD = BMq − AMq. Now



put ACq = AB × AD: and make a rectangle Triangle MAC, whose Hypotenuse CM = BM the *Semisumme* of the Sides; and the Base AM to the *Semidifference* of the Sides: the Cathetus shall be AC the Side of the Quadrat sought: by 48 e 1.

Invention of the *area* of a plaine Triangle.

Prob. V. A certain friend of mine, a learned man, brought unto me a Theoreme concerning the *area* of a plaine triangle, requesting me to examine and demonstrate it. Now the Theoreme was, as I remember (for many yeares are since passed) almost in this forme, though not in the same letters.

In a plain Triangle whose Sides are A, E, B $\left\{\begin{array}{l}\frac{1}{8}BqEq\ \frac{1}{16}Eqq\\ \frac{1}{8}EqAq-\frac{1}{16}Aqq\\ \frac{1}{8}AqBq-\frac{1}{16}Bqq\end{array}\right\}$ are equal to a quadrat of the *area* of the Triangle.

After I had a while thought upon it, there came to my minde 17. Ch. XVIII. Theor: 20. which seemed most fit for the loosing of the knot. For if the two Legges of a Triangle be A and E, and the Base B: from thence it will appeare, that $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B$ into $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B$ into $\frac{1}{2}B+\frac{1}{2}A-\frac{1}{2}E$ into $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E$, are equall to the Quadrat of the Triangles

area, multiplying therefore these Foure Magnitudes continually; there shall be brought forth $\frac{1}{8}AqEq+\frac{1}{8}AqBq+\frac{1}{8}EqBq-\frac{1}{16}Aqq-\frac{1}{16}Eqq-\frac{1}{16}Bqq$: which is the very Theoreme propounded

And from hence I not only satisfied the request; *but also afforded foure other Theoremes, more easie to be wrought.*

For because $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B=\frac{1}{2}Z+\frac{1}{2}B$.
And $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B=\frac{1}{2}Z-\frac{1}{2}B$.

And because $\frac{1}{2}B+\frac{1}{2}A-\frac{1}{2}E=\frac{1}{2}B+\frac{1}{2}X$:
And $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E=\frac{1}{2}B-\frac{1}{2}X$:
It shall be $\frac{1}{2}Z+\frac{1}{2}B$ into $\frac{1}{2}Z-\frac{1}{2}B=\frac{1}{4}Zq-\frac{1}{4}Bq$:
And $\frac{1}{2}B+\frac{1}{2}X$ into $\frac{1}{2}B-\frac{1}{2}X=\frac{1}{4}Bq-\frac{1}{4}Xq$:

It apppeares therefore first, $\frac{1}{4}Zq-\frac{1}{4}Bq$ into $\frac{1}{4}Bq-\frac{1}{4}Xq=Q$: of the *area* of the Triangle. In words thus, If a quarter of the Difference of the Quadrats of the Summe of the Legges and the Base, be drawne into a quarter of the Difference of the Quadrats of the Base and of the Difference of the Legges; the produced magnitude shall be equall to the Quadrat of the *area* of the Triangle.

Then because $\frac{1}{4}Zq-\frac{1}{4}Bq$ into $\frac{1}{4}Bq-\frac{1}{4}Xq=\frac{1}{16}ZqBq+\frac{1}{16}BqXq-\frac{1}{16}Bqq-\frac{1}{16}ZqXq$: It appeares Secondly, $Zq+Xq-Bq$ into $\frac{1}{16}Bq$ mi $\frac{1}{16}ZqXq=Q$: of the *area* of the Triangle.

Also because $Zq+Xq=2Z$ by 11, C XVIII, And $ZqXq=Xq$, by 14. C XVIII, It appears Thirdly, $2Z-Bq$ into $\frac{1}{16}Bq$ mi $Q:\frac{1}{4}X=Q$: of the *area* of the Triangle.

Lastly,

Laſtly, by theſe compared, a Fourth ariſeth $\frac{2ZBq-Bqq-Xq}{16}$ = Q: *area* Triang.

Theſe two latter Theoremes are eaſily expreſſed by words.

Probl. VI. *The Solution of Problemes about Arithmeticall Progreſſion in twenty Propoſitions.* Let the Symbols of words bee theſe: α the firſt and leaſt Terme. ω the laſt and greateſt. T the number of the Termes. X the common Difference. Z the Summe of all the Termes. Therefore T—1 is the Number of the Differences: and therefore $TX-X=\omega-\alpha$, the Summe of the Differences.

Of thoſe Five α, ω, T, X, Z, Three being given, to finde out the other two by twenty following Propoſitions (for ſo many are the varieties) in this order.

Given.	Sought.	By Propoſitions.
α, ω, T	Z and X	1 and 2
α, ω, X	T and Z	3 and 4
α, ω, Z	T and X	5 and 6
α, T, X	ω and Z	7 and 8
α, T, Z	ω and X	9 and 10
α, X, Z	ω and T	11 and 12
ω, T, X	α and Z	13 and 14
ω, T, Z	α and X	15 and 16
ω, X, Z	α and T	17 and 18
T, X, Z	α and ω	19 and 20

Prop: I. $T^{\omega} + T^{\alpha} = 2Z$.

II. $\frac{\omega - \alpha}{T-1} = X$.

III. $\frac{\omega - \alpha}{X} + 1 = T$. by 2.

IIII. $\frac{\omega q - \alpha q}{1X} + \omega + \alpha = 2Z$. by 1. 3.

V. $\frac{2Z}{\omega + \alpha} = T$. by 1.

VI. $\frac{\omega q - \alpha q}{2Z - \omega - \alpha} = X$. by 4.

VII. $TX - X + \alpha = \omega$. by 2.

VIII. $TX - X + 2\alpha$ in $T = 2Z$. by 1 and 7.

IX. $\frac{2Z - T\alpha}{T} = \omega$. by 1.

X. $\frac{2Z - 2T^{\alpha}}{Tq - T} = X$. by 8.

XI. $\sqrt{u}: \frac{1}{2}Xq + 2ZX + \alpha q - X : mi \frac{1}{2}X = \alpha$. by 4.

XII. If $B = 2\alpha - X$, it shall be
$\sqrt{u}: \frac{Bq}{4Xq} - \frac{2Z}{X} : mi \frac{B}{2X} = T$. by 8.

XIII.

XIII. $\omega+X-TX=a$. by 7.

XIV. $2\omega+X-TX$ in $T=2Z$. by 1 and 13.

XV. $\frac{2Z}{T}-\omega=a$. by 9.

XVI. $\frac{2T\omega-2Z}{Tq-T}=X$. by 14.

XVII. $\sqrt{u}: \frac{1}{4}Xq+\omega q+X\omega \left.\begin{array}{l} \\ -2ZX: \text{plus} \frac{1}{4}X \end{array}\right\}=a$. by 4.

XVIII. $\frac{2\omega+X}{2X}$ minus $\sqrt{u}: \left.\begin{array}{l} \\ \frac{4\omega q+4X\omega+Xq-2Z4X}{4Xq} \end{array}\right\}=T$. by 14.

XIX. $\frac{2Z}{2T}-\frac{TX}{2}+\frac{X}{2}=a$. by 10.

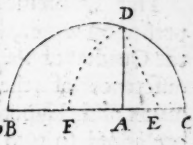
XX. $-\frac{2Z}{2T}+\frac{TX}{2}-\frac{X}{2}=\omega$. by 16.

Probl. VII. Euclid, 11 e 2. Hath taught to cut a Line given, so, that the Rectangle under the whole Line, and lesser Segment, shall be equall to the Quadrat of the greater Segment: which Section is almost divine. Let us now propose that Probleme more generally; that the Line given AB, may be so cut, that the Rectangle under the whole AB, and the lesser Segment, may, to the Quadrat of the greater Segment, have any possible Ration given: suppose of R to S.

First, put R. S::AB, AC, which here I call the Fourth Proportionall. Then for the greater Segment put A: the lesser shall be AB—A: which drawne into AB, giveth the Rectangle ABq.-AB×A. It shall be therefore AB.AC::ABq-AB×A. Aq. And therefore (by 3 Ch:VI) ABq×AC-AB×AC×A=AB×Aq. And dividing all by AB, it shall bee AB×AC-AC×A =Aq. or Aq+AC×A=AB×AC. And by 9.Ch:XVI is found √u: ½ACq+AB×AC: - ½AC=A.

This Theoreme found is thus expressed in words: If to the Quadrat of Halfe the Fourth Proportionall, be added a Rectangle under the Right line given, and that Fourth Proportionall; and out of the Quadrat-side of the Summe bee taken Halfe the fourth Proportionall: The Remainder shall bee the greater Segment.

Geome-

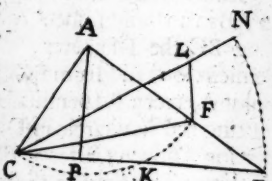


Geometrically thus: Joyne AC to AB in a ſtreight line: And making BC the Diameter, draw a Semicircle: and from BC, in the point A, erect a Perpendicular AD, cutting the Semicircle in D, then cutting AC into Halfes at E, meaſure EF = ED. I ſay, the line AB is ſo cut in the point F, that, R. S:: AB × BF. AFq. For AC × AF + AC × BF = AC × AB = ADq = CF × AF (*by* 6 e 2) = AC × AF + AFq. Wherefore AC × BF = AFq. But AB. AC:: AB × BF. AC × BF. Therefore,

Probl. VIII. If one of the Right Sides (BA or CA) of a Rightangled Triangle, in which a Perpendicular is let fall out of the Right angle unto the Hypotenuſe, be given, together with BK the Difference of the Segments of the Hypotenuſe; to finde both the Hypotenuſe, and the Triangle it ſelfe. Firſt, let there be given the leſſer Side CA. Suppoſe it done that is required: and let BAC be the Rectangled triangle: from the top whereof let downe a Perpendicular AP upon the Hypotenuſe, dividing it into the Segments BP and CP. Now $CP = \frac{BC - BK}{2}$. Becauſe BC. CA:: CA. $\frac{BC - BK}{2}$ it ſhall be $\frac{BCq - BC \times BK}{2} = CAq$: or $BCq - BK \times BC = 2CAq$. Wherefore by 9. Chap: XVI. $\sqrt{u.} \frac{1}{2}BKq + 2CAq. + \frac{1}{2}BK = BC$.

This

This Theoreme is expreſſed by words, thus; If the Quadrat of the Semidifference of the Segments of the Hypotenuſe, bee added to two Quadrats of the Side given. And the Quadrat Side of the Aggregate be increaſed with the Semidifference it ſelfe: the whole Summe ſhall be equall to the Hypotenuſe.



Geometrically thus: Draw CF: and to it a Perpendicular $FL=\frac{BK}{2}$, And extend CL to N, that $LN=\frac{1}{2}BK$. So it ſhall be $CN=BC$. Wherefore inſcribe in a Circle, $CK=CN-BK$: And produce &c. For $CFq=2CAq$. And $CLq=2CAq+\frac{1}{4}BKq$. *Ergo*,

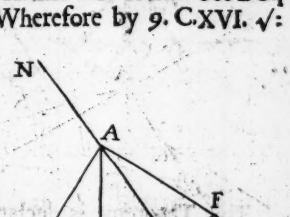
But if the greater Side BA bee given: Such an Equation ſhall bee found, $\sqrt{q:\frac{1}{4}BKq+2BAq:}-\frac{1}{2}BK=BC$. Taking $\frac{BC+BK}{2}$ For PB.

And the Geometricall Effection not unlike the former.

Probl. IX. If BF the Difference of the Sides of a Rectangle Triangle, and AP a Perpendicular from the Right Angle to the Hypotenuſe, bee given: to finde both the Hypotenuſe, and the Triangle it ſelfe.

Suppoſe it done which is deſired: and let the Rectangle

angle Triangle bee BAC. Becauſe by 7 e 2, 2BA×AF+BFq=BAq+AFq; and therefore BFq=(ABq+AFq, that is) BCq−(BA×2CA, that is) BC×2AP, becauſe BC.CA::BA.AP. It ſhall bee BCq−2AP×BC=BFq. Wherefore by 9. C.XVI. √: APq+BFq:+AP=BC.



This Theoreme is expreſſed in words thus: If to the Quadrat of the Perpendicular, be added the Quadrat of the Difference of the Sides; and the Quadrate Side of the Aggregate bee increaſed with the Perpendicular: the Totall augmented ſhall be equall to the Hypotenuſe.

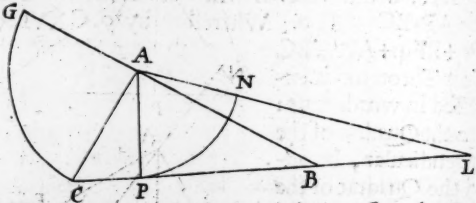
Geometrically thus: make PL=BF. And extend LA to N, ſo that AN=AP. It ſhall bee LN=BC. Therefore on the Diameter BC deſcribe a Semicircle; in which erect a Perpendicular equall to AP given. And draw BA and CA.

Probl. X. If BG the Summe of the Sides of a Rectangle Triangle, and AP a Perpendicular from the Right angle to the Hypotenuſe, be given: to finde out the Hypotenuſe, and the Triangle it ſelfe.

Suppoſe that done wch is deſired; & let the Rectangle Triangle be BAC. Becauſe by 4 e 2. BGq=(BAq+GAq, that is) BCq+(2BA×CA, that is,) 2AP×BC.

Becauſe

Becauſe BC.CA::BA.AP. It ſhall be BCq+2AP×BC =BGq. Wherefore by 9, C. XVI. √q:APq+BGq:-AP=BC.



This Theoreme is declared thus: If to the Quadrat of the Perpendicular, be added the Quadrat of the Summe of the Sides, and out of the Quadrat Side of the Aggregate, the Perpendicular be deducted: the remaining Line ſhall be equall to the Hypotenuſe.

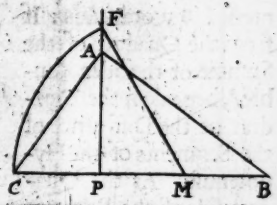
Geometrically thus: make PL =BG. And draw AL: from which cut off AN=AP. So ſhall bee LN =BC. On the Diameter therefore BC draw a Semicircle, &c.

Probl. XI. If CA one Side of a Rectangle Triangle, and BP the Alterne Segment of the Hypotenuſe, be given: to finde the other Segment, and the Triangle it ſelfe.

Imagine that effected which is deſired: And let the Rectangle Triangle be BAC. Becauſe BP+CP.CA:: CA. CP. It ſhall be BP×CP+CPq=CAq. Wherefore by 9 C.XVI, √q:½BPq+CAq:-½BP=CP.

This

This Theoreme is expreſſed in words thus: If to the Quadrat of Half the Segment of the Hypotenuſe, bee added the Quadrate of the Side given; and out of the Quadrat Side of the Aggregate, the ſame Halfe bee abated; the remaining Line, ſhall be the other Segment of the Hypotenuſe.



Geometrically thus: Set at Right angles, BP and PF=CA. And cutting BP in Halfes at M, draw MF: and meaſure MC=MF. The other Segment therefore CP is found, and alſo BC the whole Hypotenuſe. On the Diameter BC deſcribe a Semicircle; in which inſcribe CA and BA.

Probl. XII. If BK the Difference of the Hypotenuſe of a Rectangle Triangle, and BG the Summe of the Sides, be given: to finde out both the Difference of the Sides, and the Hypotenuſe, and alſo the Triangle it ſelfe.

Suppoſe that done, wch is required: and let the Rectangle Triangle be BAC. Becauſe BG. BK:: BC. BF, by 17. C. XVIII. Theor 16. And BGq. BKq:: (BCq, that is) BAq+CAq. BFq. Alſo 2BGq-BKq. BKq:: (2BAq+2CAq-BFq, that is) BGq. BFq. For by 8. C. XVIII. 2BAq+2CAq=BGq+BFq. Wherefore √q:2BGq-BKq. BG:: BK. BF:: BG. BC.

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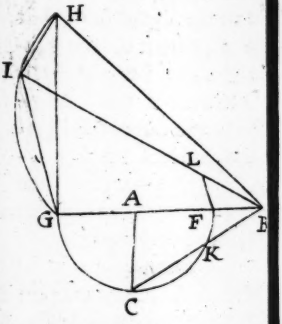


This Theoreme is expreſſed in words thus: If from the Quadrat of the Summe of the ſides doubled, bee taken the Quadrat of the Difference of the Segments of the Hypotenuſe. As the Quadrat-ſide of the Remainder is, to the Summe of the Sides; ſo ſhall the Difference of the Segments of the Hypotenuſe be, to the Difference of the Sides; and ſo the Summe of the Sides, to the Hypotenuſa.

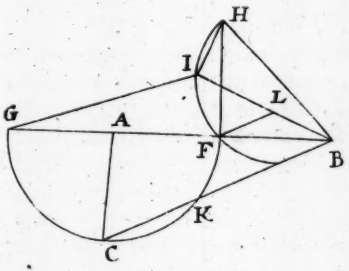
Geometrically thus: Set at Right angles, BG and GH=BG. Then on the Diameter BH deſcribe a Semicircle: in which inſcribe HI=BK: and draw BI. Therefore BI=√q: 2BGq-BKq; make alſo BL=BK. draw GI: and to it a Parallel LF. Therefore BF the Difference of the Sides, is found.

Probl. XIII. If BK the Difference of the Segments of the Hypotenuſe of a Rectangled Triangle, and BF the Difference of the Sides, be given: to finde the Summe of the Sides, and the Hypotenuſe, and alſo the Triangle it ſelfe.

Suppoſe that done which is required: and let BAC be the Rectangle Triangle. Becauſe BF. BK:: BC. BG.

by

by 17 C.XVIII Th. 16. And BFq. BKq::(BCq, that is) BAq +CAq.BGq. Alſo 2BFq - BKq. BKq:: (2BAq+2CAq-BGq, that is) BFq.BGq. For by 8. C.XVIII. 2BAq+2CAq = BGq+BFq. Wherefore √q:2BFq-BKq. BF::BK. BG.:BF.BC.



This Theoreme is expreſſed in words thus: If from the Quadrat of the Difference of the Sides doubled, be taken the Quadrat of the Difference of the Segments of the Hypotenuſe: As the Quadrat Side of the Remainder is to the Difference of the Sides; ſo ſhall the Difference of the Segments of the Hypotenuſe bee, to the Summe of the Sides; and ſo the Difference of the Sides, to the Hypotenuſe.

Geome-

Geometrically thus: Set at Right angles, BF and FH = BF. Then on the Diameter BH describe a Semicircle; in which inscribe BI = BK: and draw HI. Therefore HI = $\sqrt{q : 2BFq - BKq}$. make BL = HI. draw FL: and to it a parallel IG. Therefore BG the Summe of the Sides, is found.

Probl. XIV. If BF the Difference of the Sides of any plaine Triangle, BK the Difference of the Segments of the Base, and CL the Difference betweene the greater Side and the Base, be given: to finde the Base; the Summe of the Sides; and the Triangle it selfe. And first let the excesse be in the Base. Suppose that done which is required: and let BCD be the Triangle. Because FB. BK :: BD. $\frac{BK \times BD}{2BF}$ = BG. by 17. C. XVIII, Th. 16. It shall bee $\frac{BK \times BD - BFq}{2BF}$ = CF. Adde BF, and $\frac{BK \times BD + BFq}{2BF}$ = BC. Take this out of BD, and $\frac{2BF \times BD - BK \times BD - BFq}{2BF}$ = $\frac{2BF \times CL}{2BF}$ Wherefore $2BF \times BD - BK \times BD = 2BF \times CL + BFq$. And by 3 C. VI, 2BF — BK. 2CL + BF :: BF. BD :: BK. BG.

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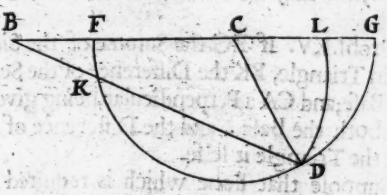

This Theoreme is expreſſed in words thus: As the Difference between the doubled Difference of the Sides, and the Difference of the Segments of the Baſe is to Aggregate of the double Difference betweene the greater Side and the Baſe, and the Difference of the Sides; ſo the Difference of the Sides, is to the Baſe; and ſo the Difference of the Segments of the Baſe, to the Summe of the Sides.

The Geometricall *Praxis* is too eaſie, to need the ſetting downe.

But if the Exceſſe be in the greater Side: the Theoreme will be BK—2BF. 2CL—BF::BF.BD:: BK.BG.

I omit the Inveſtigation of this Theoreme; and the Solution of the Probleme, wherein out of BG the Summe of the Sides of any plaine Triangle, and BK the Difference of the Segments of the Baſe, and BL the Difference betweene the greater Side and the Baſe, being given; it is required to finde out the Baſe, and the Difference of the Sides: that the Studious in

Analytice may have wherewithall to exercise their skill.

Probl. XV. If BG the Summe of the Sides of any plain Triangle, BK the Difference of the Segments of the Base; and CA a Perpendicular, being given; to finde out both the Base, and the Difference of the Sides, and the Triangle it selfe.

Suppose that done which is required: and let BCD be the Triangle. Because by 17. Ch. XVIII. Th. 16. BG. BD :: BK. BF. And by 5. Ch. XVIII, DKq = BDq + BKq − 2BK × BD. And by 47 e. 1 (4ADq, that is) DKq + 4CAq = (4CDq, that is) FGq. It shall bee BDq + BKq − 2BK × BD + 4CAq = FGq. Take FG out of BG: And BG − √q: BDq + BKq − 2BK × BD + 4CAq = BF. Wherefore it shall bee BG. BD :: BK. BG − √q: BDq + BKq − 2BK × BD + 4CAq. And by 3 Ch. VI, BK × BD = BGq − √q: BGq × BDq + BGq × BKq − BGq × 2BK × BD + BKq × BDq = BGq × BDq + BGq × BKq − BGq × 2BK × BD + BGq × 4CAq. Therefore by 8 Ch. XVI Q: BGq − BK × BD, that is, BGqq − BGq × 2BK × BD + BKq × BDq = BGq × BDq + BGq × BKq − BGq × 2BK × BD + BGq × 4CAq. And therefore BGq × BDq − BKq × BDq = BGqq − BGq × BKq − BGq × 4CAq. or also, BGq − BKq into BDq = BGq − BKq − 4CAq into BGq. Wherefore √q: BGq − BKq. √q: BGq − BKq − 4CAq :: BG. BD :: BK. BF.

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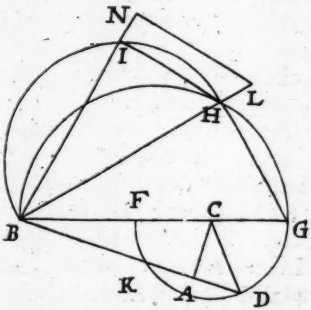
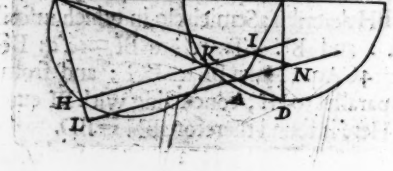


This Theoreme is expreſſed in words thus: As the quadrat Side of the Difference between the Quadrats of the Summe of the Sides, and the Difference of the Segments of the Baſe, is to the Quadrat Side of the ſame Difference leſſe by the Quadrat of the Perpendicular doubled; ſo is the Summe of the Sides to the Baſe; and ſo the Difference of the Segments of the Baſe, to the Difference of the Sides.

Geometrically thus: on BG the Diameter deſcribe a Semicircle: in which inſcribe GH = BK: and BH. Therefore BH = √q: BGq - BKq. Again on the Diameter BH deſcribe a Semicircle; in which inſcribe HI = 2CA: and BI. Therefore BI = √q: BGq — BKq — 4CAq. Make BL = BG: and from L draw LN parallel to HI, concurring with BI extended at N. Here is found therefore BN = BD.

Probl. XVI. If BF the Difference of the Sides of any plaine Triangle, BK the Difference of the Segments of the base, and CA a Perpendicular, be given: to finde both the base, and the Summe of the Sides, and also the Triangle it selfe.

Suppose that done which is required: and let BCD bee the Triangle. Because BG.BD::BK.BF. And DKq = BDq + BKq - 2BK×BD, by 5 Ch. XVIII. And by 47 e 1, (4ADq, that is) DKq + 4CAq = (4CDq, that is) FGq. It shall bee BDq + BKq — 2BK×BD + 4CAq = FGq. Adde FG to BF: And BF + √q: BDq + BKq - 2BK×BD + 4CAq = BG. Wherefore BF.BD::BK.BF + √q:BDq + BKq - 2BK×BD + 4CAq. Also BK×BD = BFq + √q: BFq×BDq + BFq×BKq - BFq×2BK×BD + BFq×4CAq. Therefore Q:BK×BD - BFq, that is, BKq×BDq - BFq×2BK×BD + BFqq = BFq × BDq + BFq×BKq - BFq×2BK×BD + BFq ×4CAq. And therefore BKq×BDq - BFq×BDq = BFq×BKq - BFqq + BFq×4CAq. or also BKq - BFq into BDq = BKq - BFq + 4CAq into BFq. Therefore √q:BKq - BFq. √q: BKq - BFq + 4CAq :: BF.BD::BK.BG.



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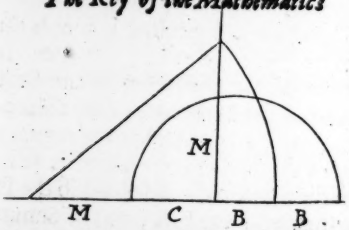
This Theoreme is expreſſed in words thus: As the Quadrat Side of the Difference betweene the Quadrats of the Difference of the Segments of the baſe, and of the Difference of the Sides, is to the Quadrat Side of the ſame Difference augmented with a Quadrat of the double Perpendicular, ſo is the Difference of the Sides, to the Baſe; and ſo the Difference of the Segments of the Baſe, to the Summe of the Sides.

Geometrically thus: on BK the Diameter deſcribe a Semicircle: in which inſcribe KH = BF and BH. Therefore BH = √q: BKq - BFq. Make BHL = BF: and HKI = 2CA. Draw BI. Therefore BI = √q: BKq — BFq + 4CAq. Draw alſo LN Parallel to HI, concurring with BI extended in the Point N. Here is therefore found BN = BD.

Probl. XVII. If in a Rectangle Triangle, B the Difference between the Baſe and Hypotenuſe, & C the Difference between the Cathetus and Hypotenuſe: to finde both the Hypotenuſe and the Triangle it ſelfe.

For the Hypotenuſe put A. the Baſe will be A — B. and the Cathetus A — C. and by 47 e 1, the Cathetus is √q: 2BA — Bq. Wherefore √q: 2BA — Bq = A — C. And 2BA - Bq = Aq — 2CA + Cq. or 2B + 2C into A mi Aq = Bq + Cq. Therefore by 9 Ch. XVI, B + C + √q: 2BC = A the Hypotenuſe.



This Theoreme is expreſſed in words thus: the Aggregate of the two Differences, that is, the Difference both of the Baſe and of the Cathetus, from the Hypotenuſe, together with the Quadrat Side of a double Rectangle under the Differences, is equall to the Hypotenuſe.

Geometrically thus: Draw an infinite Line, in which meaſure B, B, and C. This being taken for the Diameter, draw a Semicircle; and in the common point of B and C, erect Perpendicularly the Line M. therefore Mq=2BC. Alſo from C in the infinite Line, meaſure M, and with the Semidiameter M+C+B, deſcribe an Arch, untill it concurre with the Perpendicular Line M extended. Then from the point of concourſe to the Center of that Arch, draw a Line for an Hypotenuſe. And ſo the deſired Rectangle Triangle ſhall be deſcribed.

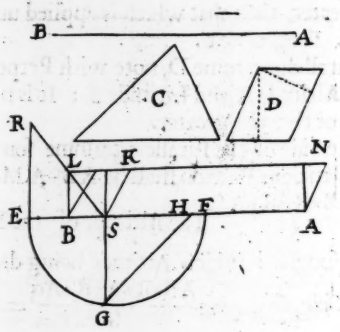
Probl. XVIII. Unto a Line A B given, to apply a Parallelogramme equall to a plain figure C given: which Parallelogramme ſhall fall ſhort of the ſaid right Line, by a Parallelogramme Figure like unto D a Parallelogramme aſſigned. But the right lined Figure C muſt not

not be greater, then that which is applied unto halfe that Line. Prop. 28. e 6.

In a Parallelogramme D, note with Perpendicular Lines its Altitude R, and Latitude S : It is no matter whether of them be greater.

For the Side of the Parallelogramme ſought, put A: the ablatitious Portion ſhall be AB–A. Make S.R:: AB–A. $\frac{AB*R-R*A}{S}$. The Altitude of the Parallelogramme ſought : which Altitude being drawne into A the Side, ſhall give $\frac{AB*R*A-R*Aq}{S} = C$: or $AB*A-Aq = \frac{C*S}{R}$: And by 9 Ch. XVI, $\frac{AB}{2} \pm \sqrt{q}$: $\frac{ABq}{4} - \frac{C*S}{R} := A$.

Which Theoreme is expreſſed in words thus : If C the plaine Figure given, be drawne into the Latitude of the Parallelogramme D; and the Product bee divided by the Altitude; and the Quotient taken out of Halfe the given Line AB : the Quadrate Side of the Remainder increaſed with the ſame Halfe, ſhall be the Side of the Parallelogramme ſought.



Geometrically thus: make ER $=\sqrt{qC}$. Then R.S::ER.ES $=\frac{\sqrt{qC}}{R}$. Set ER and ES at Right angles: and taking SF $=$ ER, or on the Diameter EF describe a Semicircle: in which erecting a Perpendicular SG, there will be SGq $=\frac{C*S}{R}$. From the point G measure GH $=\frac{1}{2}$AB $=$ HB: It will bee HS $=\sqrt{:\frac{ABq}{4}-\frac{C*S}{R}}$: to which if you adde HA $=\frac{1}{2}$AB; you shall have AS the Side of the Parallelogramme sought. And BS $=$ BA $-$ A the ablatitious Portion. And BL parallel to the Line ER, shall be the Altitude. Therefore the Parallelogramme sought is ASKN, being made Equiangle with D.

Probl.

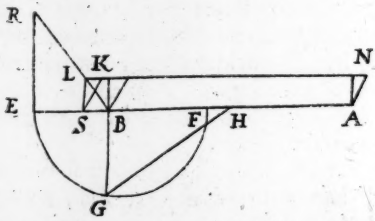
Probl. XIX. Unto a right Line AB given, to apply a Parallelogramme equall to a plaine Figure C given: which Parallelogramme ſhall overreach the ſaid Right line, by a Parallelogramme Figure like unto D a Parallelogramme aſſigned. Prop. 29. & 6.

In the Parallelogramme D note the Altitude and Latitude, as in the former.

For the Side of the Parallelogramme ſought put A: the adjectitious Portion ſhall be A-AB, Make S.R:: A-AB. $\frac{R \times A - AB \times R}{S}$ the Altitude of the Parallelogramme ſought: which Altitude being drawne into A the Side, it ſhal give $\frac{R \times Aq - AB \times R \times A}{S} = C$. or $Aq - AB \times A = \frac{C \times S}{R}$. And by 9 Ch. XVI, $\sqrt{q}: \frac{ABq}{4} + \frac{C \times S}{R}: + \frac{AB}{2} = A$.

Which Theoreme is expreſſed in words thus: If C the plaine Figure given, be drawne into the Latitude of the Parallelogramme D; and the Product bee divided by the Altitude; and the Quotient added to the Quadrat of Halfe the Line given AB: The Quadrat Side of the Aggregate increaſed with the ſame Halfe, ſhall be the Side of the Parallelogramme ſought.

Geometri-



Geometrically thus: Make ER=$\sqrt{qC}$. Then R.S::ER.EB=$\frac{\sqrt{C*S}}{R}$. Set ER and EB at Right angles: and taking BF=ER, on the Diameter EF describe a Semicircle: in which erecting a Perpendicular BG, there shall be BGq=$\frac{C*S}{R}$. Measure BH =$\frac{1}{2}$AB=AH. And draw GH=$\sqrt{q: \frac{ABq}{4} + \frac{C*S}{R}}$: =HS. Therefore AS=A the Side of the Parallelogramme sought. And BS=A-AB the adjectitious Portion. And the Altitude shall be SL Parallel to the Line ER. Therefore the Parallelogramme sought is ASKN, being made Equianguler with D.

Probl. XX. If BC, BD, two Sides of any plaine Triangle, with B the Angle comprehended betweene them, be given; to finde out the Third Side: or if the Three Sides being given; to find out the Angle B opposite to one of them.

Suppose

Suppoſe that done which is required: and let BCD be the Triangle. On the Center B, and Semidiameter BC, deſcribe an Arch CK, and let fall a Perpendicular CA. Therefore KD is the Difference of the Sides: and AK like to the verſed Sine of the Angle B. For Rad. *sv*B::BK.AK$=\frac{BK\times sv B}{Rad}$. But AK$=BK\mp$BA; as appeareth by the Schemes compared.

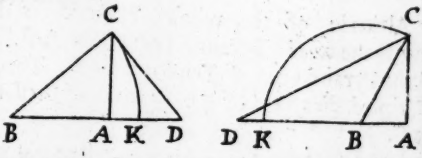
And becauſe $\left.\begin{matrix}?\\?\end{matrix}\right\}=\left\{\begin{matrix}2BD\times BK+KDq.\\CDq\mp 2BD\times BA.\end{matrix}\right.$ by 5 Ch. XVIII. by 2,3, Ch. XIX.

It ſhall bee, $2BD\times BK+DKq=CDq\mp 2BD\times BA$.

Wherefore $2BD\times BK\mp 2BD\times BA$, that is, $2BD\times AK+DKq=CDq$. But $2BD\times AK=\frac{2BD\times BC\times sv B}{Rad.}$

Therefore $\frac{2BD\times BC\times sv B}{Rad.}+DKq=CDq$. which is the firſt Theoreme.

And $\frac{CDq-KDq \text{ into } Rad}{2BD\times BC}=sv B$. which is the ſecond Theoreme.



The firſt of theſe Theoremes is expreſſed in words thus: If a doubled Rectangle contained under the Sides

Sides given, be drawne into the *verſed Sine* of the Angle intercepted; and the Product be divided by the *Radius*: The Quotient increaſed with a Quadrat of the Difference of the Sides ſhall be equall to the Quadrat of the third Side.

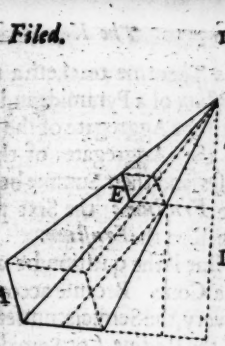
And the Second thus: If the Difference of the Quadrats of the oppoſite Side, and of the Difference of the Sides be drawne into the *Radius*; and the Product be divided by a doubled Rectangle under the containing Sides: the Quotient ſhall be equall to the *verſed Sine* of the Angle ſought.

Probl. XXI. In a piece or *Fruſtum* of a Pyramid, both the Baſes Aq, Eq, and the Altitude L, being given; to finde the meaſure of the *Fruſtum*.

You muſt firſt know (out of 7 and 10 e 12) that a *Parallepipedon* is equall to three Pyramids; and a Cylinder to three Cones of the ſame Baſe and Altitude.

The Altitude T of the Pyramid cut off, muſt firſt be ſought, thus; A-E.E::L.T. Wherefore $\frac{LE}{X}=T$. And the Altitude of the whole Pyramid is L+T. Likewiſe the whole Pyramid Tripled is AqL+AqT. And the Pyramid cut off Tripled is EqT. Therefore the Tripled *Fruſtum* of the Pyramid is AqL+AqT -EqT.

This



This Theoreme ſheweth one way of meaſuring the *Fruſtum* of a Pyramid: And is expreſſed in words thus: If the Solid under the greater Baſe and whole Altitude, be leſſened by a Solid under the leſſer Baſe and Altitude of the Pyramid cut off: the third part of the Remainder ſhall be equall to the *Fruſtum*.

Againe, becauſe by 2 Ch.XI, Aq-Eq=ZX: and $T=\frac{I.E}{X}$. It ſhall be AqL+ (ZEL, that is, by 3 e 2) AEL+EqL=AqL+AqT-EqT. Therefore the tripled *Fruſtum* of the Pyramid is Aq+Eq+AE into L.

This Theoreme teacheth another way of meaſuring the *Fruſtum* of a Pyramid: and it is expreſſed in words thus:

If the Aggregate of both the Baſes of the *Fruſtum* of a Pyramid, and of the Meane-Proportionall between them, be drawne into the Altitude of the *Fruſtum*: the third part of the Product ſhall bee equall to the *Fruſtum*.

Alſo becauſe, by 2 Ch.XI. 2Aq+2Eq=Zq+Xq: ZqL+XqL+2AEL ſhall be equall to ſix *Fruſta's*. But by 11 Ch.XVIII. Xq+2AE=Z. Therefore Zq+Z into L is equall to ſix times the *fruſtum* of a Pyramid. And this

this Theorme teacheth a third way of meaſuring a *fruſtum* of a Pyramid; and is expreſſed in words thus: If to the Aggregate of the Baſes be added the Quadrat of the Aggregate of the Quadrat-Sides of both Baſes, and their Summe be drawn into the Altitude of the *Fruſtum*: the Sixt part of the Product ſhall be equall to the *Fruſtum*.

But if the queſtion be about meaſuring a *Fruſtum* of a Cone. Becauſe according to *Archimedes* his diſcovery, the Semicircumference of a Circle is equall $\frac{22}{7}$ of the Radius (or Semidiameter) almoſt: or more accurately $\frac{355}{113}$ Rad. The *area* of a Circle ſhall be $\frac{355}{113}$ Rad:q. And 113. 355::Rad:q. *area* Circle.

Wherefore the firſt Theoreme of meaſuring the *Fruſtum* of a Cone, is $\frac{355}{452}$ AqL+$\frac{355}{452}$AqT-$\frac{355}{452}$EqT, are equall to the *Fruſtum* tripled.

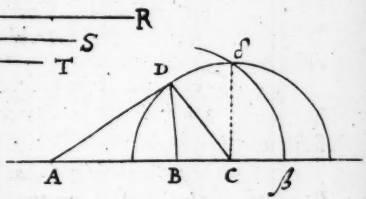
The Second is $\frac{355}{452}$ Aq+ $\frac{355}{452}$ Eq+ $\frac{355}{452}$ AE into L, are equall to the *Fruſtum* tripled.

The third is $\frac{355}{452}$ Zq+ $\frac{355}{452}$ Z into L, are equall to the *Fruſtum* of the Cone Six-fold.

Probl. XXII. A Probleme of *Apollonius Pergaus* ἐν ἀναλυόμενῳ τόπῳ. Two points A, B, being given in a Plaine, to deſcribe a Circle, to whoſe Circumference, the Right lines AD, BD, drawn from the ſame Points, may have the Ration given of R to S.

Suppoſe that done which is required: and let the Center of the Circle ſought be C in the ſame Right line with the Points A, B; and the Semidiameter CD. Make R.S::S.T. Now becauſe the two Triangles ACD,

ACD,DCB (whereſoever the Point D be taken) are as AC to BC: And the Sides DA,DB, alike oppoſite to the common Angle C, are in the Ration of R to S: And the Side CD common to both: one may eaſily conceive the Triangles ACD,DCB to be like. Wherefore R.S::DA.DB::AC.DC::DC.BC. And by 1 Ch. XV, AC.BC::Rq.Sq::R.T. If therefore for BC bee put A: It ſhall be AB+A.A::R.T. And $AB*T+T*A=R*A$: or $\frac{AB*T}{R-T}=A$. Laſtly, $\sqrt{:AC*BC:}=DC$.



Which are expreſſed in words thus: If the ſpace betweene the points, be drawn into the third Proportionall Terme of the Ration given: and the Product bee divided by the Exceſſe of the Firſt Terme above the Third: The Quotient ſhall be equall to the Diſtance of the nearer Point from the Center. And the Quadrat-ſide of the Rectangle under both Diſtances from the Center ſhall be equall to the Semidiameter. The Geometricall Effection is eaſie.

Probl. XXIII. In a Tunne or Wine-veſſell, the interne Longitude 2CL; and the two Semidiameters, Name-

namely CB at the Bung, and LD at the Head, being given: to finde out the capacity of the Tunne it self. A Tun is a *Fruſtum* of a *Sphæroides*, which Solid figure is made by the Revolution of Halfe an *Ellipſis* (or Ovale) about its tranſverſe Diameter or *Axis*. Now that the meaſure of the *Fruſtum* may be found out, the meaſures both of the whole *Spheroides* and of the Portions cut off, muſt be knowne: for the Difference of theſe meaſures is the meaſure of the *Fruſtum*.

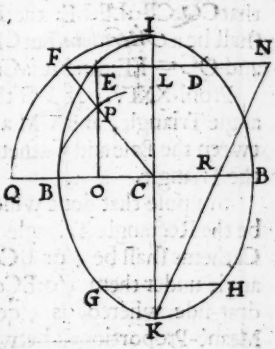
The Solidity of the whole *Spheroides*, is $\frac{355}{113}$ BCq into $\frac{2}{3}$ IK which is double a Cone made on the Baſe BCB, at the Altitude IK. *Archim de conid. & Sphæroid.* Prop. 29.

And the Solidity of IED the Portion cut off, is had thus: LK.LK+KC::$\frac{355}{113}$ LDq into $\frac{1}{3}$ LI. Solidity ſought. *Ibid.* Prop. 31.

But here is yet wanting (that which is the maine Point of this buſineſſe) the tranſverſe Diameter axe IK: which you ſhall thus finde.

Suppoſe that done which is required: And deſcribe an *Ellipſis*: &c. All elſe as in the Scheme. And make CK.CB::CB.$\frac{CBq}{CK}$=CR, Halfe the right *Latus* by 13 *l* 11 *Conic: Apoll.* Againe make CK.$\frac{CBq}{CK}$::CK+CL.$\frac{CBq \text{ into } CK+CL}{CKq}$=LN. Draw it into IL, that is, CK-CL (which is all one as if you draw CBq into CKq-CLq. by 11 Ch: 18. and there ſhall ariſe CBq

$\frac{CBq \times CKq - CBq \times CLq}{CKq} = LEq$, by 13, l. 1. Conic: *Apoll*: Therefore $\sqrt{q: \frac{CBq \times CLq}{CBq - LEq}} = CK$, the Semi-axe; that is, $\frac{CB \times CL}{OP} = CK$.



Which Theoreme is expressed in words thus: If the Quadrat of the Semidiameter of the middle of the Tun, bee drawne into the Quadrat of Halfe the Length: and the Product bee divided by the Difference between the Quadrats of the Semidiameters of the Middle and Base: the Quadrat side of the Quotient shall be the Semi-axe of the *Spheroides*.

Geometrically thus: Draw EO Parallel to the Axe. And on the Semidiameter CP = CB make an Arch cutting EO at P. extend CP untill it concurre with the Base LE extended at F. And CF shall be equall to the Semi-axe sought.

Consectarie. From hence it appeares the Meridians in the *Analemma* are true *Ellipses*. For example,

Suppose CIFQ a Quadrant of the *Analemma*; in which is described an *Ellipsis* IEB. I say, the same is a *Meridian*. For seeing that CQ is a Quadrant of the Æquinoctiall and FL a quadrant of its Parallel: and that it is the property of the *Meridians* to cut the Æquinoctiall, and all its Parallel Circles, into like Segments by 10.l.2.*Theod: de Spher:* If therefore it be evince that CQ.CB::LF.LE: the *Ellipsis* IEB cutting them shall be a *Meridian*. But CF=CQ: and CP=CB and OC=LE: And it is CF.CP::LF.OC. Therefore

Probl.XXIV. If BC the Hypotenuse of a Rectangle Triangle, and CM a Meane-Proportionall between the Base and Cathetus, be given; to finde out the Triangle.

Suppose that done which is desired: and let BA be the Rectangle Triangle. Because the Base is BA, the Cathetus shall be $\sqrt{q:BCq-BAq}$: And the Rectangle under them $\sqrt{q:BCq\times BAq-BAqq}$: the Quadrat-side whereof is $\sqrt{qq:BCq\times BAq-BAqq}$: the Meane-Proportionall between the Base and Cathetus.

Againe because the Cathetus is CA, the Base shall be $\sqrt{q:BCq-CAq}$: And the Rectangle under them $\sqrt{q:BCq\times CAq-CAqq}$: the Quadrat-side whereof $\sqrt{qq:BCq\times CAq-CAqq}$: the Meane-Proportionall between the Base and Cathetus. Wherefore

$BCq\times BAq-BAqq=CMqq$. And

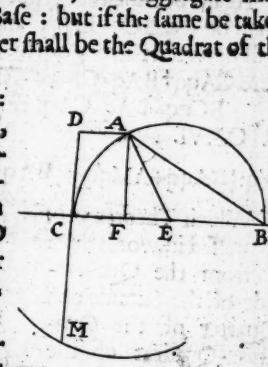
$BCq\times CAq-CAqq=CMqq$. Therefore by 9 Ch.X

$$\tfrac{1}{2}BCq\pm\sqrt{q:\tfrac{1}{4}BCqq-CMqq:}=\begin{cases}BAq\\CAq\end{cases}$$

Which Theoreme is expressed in words thus:

to Halfe the Quadrat of the Hypotenuſe, be added, the Quadrat-ſide of the Exceſſe of a Quarter of the Quadrato-quadrat of the Hypotenuſe above the Quadrato-quadrat of the Meane-Proportionall betweene the Baſe and Cathetus; the Aggregate ſhall be the Quadrat of the Baſe: but if the ſame be taken out of it; the Remainder ſhall be the Quadrat of the Cathetus.

Geometrically thus: on the Diameter BC, and Center E the middle point thereof, deſcribe a Semicircle: then make BC.CM::CM.CD =AF a Perpendicular within the Semicircle. Therefore BC*AF= CMq. make up the Triangle BAC. For ½BCq (AEq)-AFq=EFq. Wherefore.



$$\tfrac{1}{2}BC \pm (EF)\sqrt{q:\tfrac{1}{4}BCq-AFq.} = \begin{cases} BF \\ CF \end{cases}$$

Draw all into BC: and the Product ſhall be

$$\tfrac{1}{2}BCq \pm \sqrt{q:\tfrac{1}{4}BCqq-(BCq*AFq)CMqq:} = \begin{cases} BC*BF=BAq \\ BC*CF=BAq \end{cases}$$

Probl. XXV. If BA the Baſe of a Rectangle triangle, & AM a Mean-Proportionall between the Hypotenuſe and Cathetus, be given: to find out the Triangle.

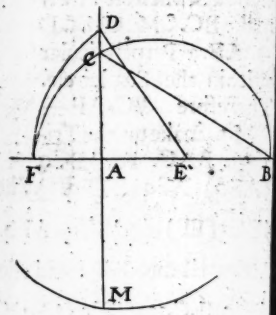
Suppoſe that done which is deſired: and let BAC be

the Rectangle Triangle. Because the Cathetus is CA, the Hypotenuse shall bee $\sqrt{q}$:BAq+CAq : and the Meane-Proportionall betweene them, is $\sqrt{qq}$: CAqq +BAq×CAq:

Again because the Hypotenuse is BC, the Cathetus shall be $\sqrt{q}$:BCq−BAq : and the Meane-Proportionall between them $\sqrt{qq}$:BCqq−BAq×BCq: Wherefore CAqq+BAq×CAq=AMqq. And

BCqq−BAq×CAq =AMqq. Therefore by 9. C.XVI.

$$\sqrt{q}:\tfrac{1}{4}BAqq+AMqq:\mp\tfrac{1}{2}BAq=\begin{cases}CAq.\\BCq.\end{cases}$$



Which Theoreme is expressed in words thus: If from the Quadrat-side of the Summe of a Quarter of the Quadrato-Quadrat of the Base, and of the Quadrato-quadrate of a Meane Proportionall betweene the Hypotenuse and Cathetus, bee taken the Half Quadrate of the Base; the Remainder shall bee the Quadrat of the Cathetus : But if the same be added to it the Aggregate shall be the Quadrat of the Hypotenuse.

Geome-

Geometrically thus : Make BA.AM::AM.AD Perpendic : Therefore BA×AD=MAq. From the Point E the middle of the Base to the Perpendicular AD, draw ED=EF. And on the Diameter BF describe a Semicircle cutting AD in C. Then drawing BC, make up the Triangle BAC. For $\frac{1}{4}$BAq+ADq =EFq. Wherefore,

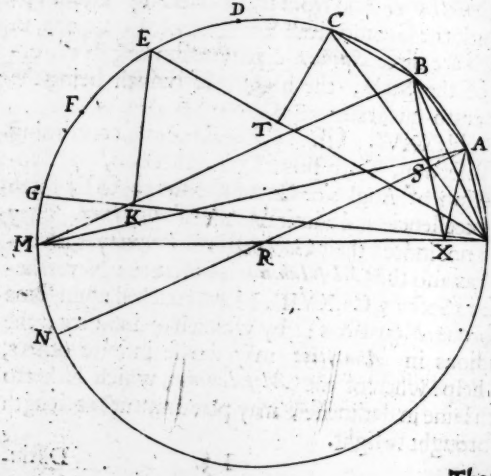
$\sqrt{q:\frac{1}{4}BAq+ADq:\mp\frac{1}{2}BA} = \frac{AF.}{BF.}$ Draw all into BA:& it shall be $\sqrt{q:\frac{1}{4}BAqq+(BAq\times ADq,}$ that is) $Mqq:\mp\frac{1}{2}$

$BAq = \begin{cases} BA\times AF = CAq. \\ BA\times BF = BCq. \end{cases}$

Consectarie. And from these two Propositions appeareth the Geometricall Effection of Æquations, in which are three *Species* equally ascending in the order of the Scale, the highest of which being the Quadrato-quadratic.

Probl: XXVI. I shall also adde a little concerning the Bisection, Trisection, Quinquisection, and Sepisection of Angles or Circumferences, to set forth the excellencie and admirable use of *Analytice*. They have not indeed their Geometricall *Praxis* yet invented : as also the *Mesolabium* is unfound : Nevertheesse in Sect 15, Ch, XVIII, I have touched upon some Cubique Æquations; by which like industry, the Studious in *Analytice* may devise infinite others, by helpe whereof the *Mesolabium*, which hitherto hath laine in darknesse, may peradventure at length be brought to light.

Distinguish in a Circumference Seven equall parts from O the end of the Diameter, with letters ABCDEFG: draw the subtenses as in the Scheme. Take MX=MB. Draw also AX and XB; and a Diameter NRA; and CT a Perpendicular to OE. Now because by 17 Ch: XVIII. Th: 1. AB=AX: the Triangles BMX, ORA, OAX, shall be Like, and therefore $\frac{OAq}{Rad.}$=OX. Likewise the Triangles OAB, ARM, are Like. And by 47 e 1. MA=√q: 4Radq -OAq.



These

Theſe thus premiſed, it ſhall be RA.MA, that is, Rad. $\sqrt{q}$: 4 Radq − OAq :: OA. OB. Therefore $\frac{4Radq \times OAq - OAqq}{Radq} = OBq$: which is the Duplication of an Angle.

And 4 Radq × OAq − OAqq = Radq × OBq: which is the Biſection of an Angle.

Then becauſe OS = OA. And SA = OX. And NS = MX = MB. It ſhall be by 17, Ch. XVIII. Th. 16. $\frac{NS \times SA}{OS} = SC$, that is, $2\,Rad - \frac{OAq}{Rad}$ into $\frac{OAq}{Rad}$ divided by OA, or $\frac{2Radq \times OA - OAc}{Radq} = SC$. And if OA be added, then ſhall be made $\frac{3Radq \times OA - OAc}{Radq} =$ OC: which is the Triplication of an Angle.

And 3Radq × OA − OAc = Radq × OC: which is the Triſection of an Angle.

Alſo becauſe 2ET + CB = OE. And MO.MB :: OC. OT: that is, $2Rad.2Rad - \frac{OAq}{Rad} :: \frac{3Radq \times OA - OAc.}{Radq}$ $\frac{6\,Radqq \times OA - 5Radq \times OAc + OAqc}{2\,Radqq}$: Out of the Double whereof if OA be taken; there ſhall remaine $\frac{5Radqq \times OA - 5Radq \times OAc + OAqc}{Radqq} = OE$: which is the Quintuplation of an Angle. And OAqc − 5Radq × OAc + 5Radqq × OA = Radqq × OE:

which is the Quinquiſection of an Angle.

And in this forme we may proceed to finde the Septiſection, namely $7Rcc\times OA - 14Rqq\times OAc + 7Rq\times OAqc + OAqqc = Rcc\times OG$. For MO. MB:: OE. OK. And $2OK - OC = OG$. The Operation I leave to the Studious.

Now becauſe the *Radius* is put 1, which in Multiplication and Diviſion, alters nothing: therefore in all theſe Æquations, the *Radius* with all its *Poteſtates* may be omitted.

But by what Artifice thoſe elaborate Æquations (in which there are not only three *Species* equally aſcending in the order of the Scale) may be reſolved, though it belong not to this preſent purpoſe to teach; yet what I have commented and written to the uſe of the Right Honourable and learned Lord *Gerard Aungier*, Lord *Aungier* and Baron of *Longford*, very many yeares agoe, I will make bold, for the benefit of the Studious in the *Mathematics*, with all poſſible brevity, to communicate.

OF

OF THE RESOLVTION OF ADFECTED ÆQUATIONS IN NUMBERS.

1. THE manner of conſtituting an Adfected Æquation. Take, as you pleaſe, for B, 3: For Cq, 16: 1or Dc, 125: for Fqq, 1296:&c. (and it makes no matter whether the numbers taken,be truly figurate or no.) And of theſe Coefficients let there bee made a Quadrato-cubic Æquation. It being framed after the manner of the latter Analyticall Table, in the Quadrato-quadratic row, ſhall be Lqc-5BLqq+10CqLc-10DcLq+5FqqL=Gqc. which in numbers putting L (the root) 47, ſhall be 1qc-15qq+160c-1250q+64801 = 170304782. Or omitting the diſtinction of the *uncia*, for 15qq, ſay BLqq; for 160c, ſay CqLc; for 1250q, ſay DcLq; and for 64801, ſay FqqL. For if L bee 47: then is Lq =2209: and Lc=103823:and Lqq=4879681:and Lqc=229345007.

The

The Practice of this Construction.

BL qq	229345007
15 × 4879681	−73195215
CqLc	156149792
160 × 103823	+16611680
DcLq	172761472
1250 × 2209	−2761250
FqqL	170000222
6480×47	+ 304560
	170304782

2. Propose any Æquation whatsoever; if you please, that now found,
1qc−15qq+160c−1250q+6480l = 170304782: or changing the numbers into Symbols,
Lqc−BLqq+CqLc−DcLq+FqqL=Gqc:

And if there were more *Species* of Adfections, they might consequently be expressed by Hcc, Kqqc, Mqcc, Nccc, and so on further.

3. Of the Root L required to bee found out of these, there are two parts; Namely, A the First Side, and E the second, or any Side following.

Wherefore L=A+E: and all the *Potestates* of L, are equall to the like *Potestates* of A+E: as Lq=Aq +2AE+Eq: and Lc=Ac+3AqE+3AEq+Ec.&c.

Hee therefore that desires to learne this numerous re-

reſolution of Adfected *Poteſtates*, muſt be well verſed in the Geneſis and Analyſis of Pure *Poteſtates*.

4. In the propoſed Æquation, the *Poteſtas* to bee reſolved, *viz.*: 17030478 2, or Gqc is Quadrato-cubic: of which *kinds* alſo are all the other ſeverall *Species* of the Adfections. For Heterogeneals can neither be added one to another, nor ſubſtracted.

5. Wherefore in the ſeverall Adfections, two things are to be conſidered; the degree of the Adfection, and the Coefficient: As in 15qq the Degree of Adfection is Quadrato-quadratic, and the Coefficient 15 is Lateral: In 160 c, the Degree of Adfection is Cubic, and the Coefficient 160 Quadratic: In 1250q, the Degree of Adfection is Quadratic, and the Coefficient 1250 is Cubic: Laſtly, in 6480 l, the Degree of Adfection is laterall, and the Coefficient 6480, Quadrato-quadratic: As will evidently appeare by comparing both the deſignations of that Æquation. And from hence ariſe two *Conſectaries* for the extraction of the ſingle Roots.

6. The firſt *Conſectarie* is, If the Root of the Coefficient for its owne *kinde*, drawne into the Degree of Adfection, be multiplyed by the Coefficient: the Product ſhall be of the ſame *kinde* with the *Poteſtas* Reſolvend: As in the precedent Æquation, If the Side 15 multiplied Quadrato-quadraticly, be drawn into 25; and if √q160 cubed, be drawne into the Quadrat 160; and if √c 1250 quadrated, be drawne into the Cube 1250; Laſtly, if √qq 6480, bee drawne

drawne into the Quadrato-quadrat 6480; Out of every one of theſe ſeverall Multiplications, ſhall ariſe a Quadrato-cubic number. And this *Analyticall Multiplication is the way of reducing any Coefficient to the Species of the Poteſtas Reſolvend*, moſt uſuall in the extraction of the firſt ſingle Root.

7. Whence alſo it evidently appeareth, that if the number ariſing out of the Coefficients reduced in this manner and compared, be leſſe than the Side of the *Poteſtas* Reſolvend, his Side alſo is leſſe than the Side of the *Poteſtas* Reſolvend: but if greater, it is greater: and if equall, equall. Therefore in this Æquation $1qc-15qq+160c-1250q+64801 = 17030478^2$: or $17030478^2+15qq-160c+1250q-64801 = 1qc$, if both the laterall Coefficient 15, and alſo $\sqrt{q160}$, and alſo $\sqrt{c1250}$, and laſtly, $\sqrt{qq6480}$, be Quadrato-cubed; there ſhall come forth foure Homogene *Species* of Adfections, namely 7593.., 3238.., 1450.., 0581.., Which indeed is eaſily done by Logarithmes, and exactly enough for the purpoſe. The manner of working it (to be fetched out of the end of this Tract, where ſome rules are delivered about the knowledge of Logarithmes) is thus:

Lo-

Logarithmes.	Coefficients.
5 × 1,17609	15 qq
5,88045	7593..
2) 2,20412	160 c
5 × 1,10206	
5,51030	3238..
3) 3,09691	1250 q
5 × 1,03230	
5,16150	1450..
4) 3,81157	6480 l
5 × 0,95289	
4,76445	0581..

In the Æquation therefore proposed, the *Species* according to their respective signes, being gathered into one Summe; it shall bee 170304700+759300 —32300+145000 — 058100 = 1qc = 170827100. which may be done in like manner also in all other Æquations.

The second *Consectarie* is, If the *Potestas* Resolvend be divided by the Coefficient; the Quotient shall be referred to the same degree of Adfection: that is, the Quotient shall be a Side, if the Adfection be under

der a Side; or a Quadrat, if under a Quadrat; and ſo of the other degrees: As in the former Æquation, If 17030478 2 be divided by 15, the Quotient ſhall be Quadrato-quadratic; if by 160, the Quotient ſhall be Cubic; if by 1250, the Quotient ſhall bee Quadratic; laſtly, if by 6480, the Quotient ſhall be laterall. Wherefore not alwaies the Quotient it ſelf, but moſt commonly its Side for the degree of Adfection reſpectively, ſhall be the ſingle Side to be extracted.

9. In the Inveſtigation alſo of the ſecond Side, this muſt be held; that as the number of Figures in the Quotient is, ſo is its degree moſt commonly to be valued: as if the Quotient conſiſt of one Figure only, it is a Side; if of two, a Quadrat; if of three, a Cube; &c. And if the Quotient exceed 5, or 50, or 500, &c. It may peradventure be extended to the following degree, eſpecially in greater Adfections. And *theſe are the Lawes of Analyticall Diviſion.*

10. Neither will it be needfull in this *Multiplication* and *Diviſion* to runne through the whole *Poteſtas* Reſolvend, with the whole Coefficient, but only as far as the next congruent Point.

11. For in the reſolution of Æquations adfected, all the Punctations of the degrees muſt bee made in the *Poteſtas* Reſolvend, as they are in the Pure. Thoſe of the higheſt degree above; and thoſe of the other degrees, beneath. The Coefficients alſo, each for their *Species* reſpectively, are to be pointed. The Punctations of the former example ſhall be thus:

$$1qc - \overset{\cdot}{1}\overset{\cdot}{5}qq + \overset{\cdot}{1}\overset{\cdot}{6}0c - \overset{\cdot}{1}\overset{\cdot}{2}\overset{\cdot}{5}0q + \overset{\cdot}{0}\overset{\cdot}{6}\overset{\cdot}{4}\overset{\cdot}{8}0l = 170\dot{3}0478\dot{2}$$

12. The number of the Points (especially if the Coefficient be negative) shall regularly in each *kinde* be equall. Wherefore if the *Potestas* Resolvend happen to have more or fewer Points above it, than the Coefficient hath, so many Circles are to bee set before the number defective, that the Points in each may be equall. And in extracting the severall Sides, that Point of the Coefficient which is proper to that Side, shall be set under the like Point of the *Potestas* resolvend: which shall well be done, if the unit place in the Coefficient be orderly set backe unto the lower Points of the *Potestas*, answering to the degree.

13. If any Coefficient be a Fraction or Surde Side, let it be reduced to integres, with Decimall parts.

14. And if it be needfull to continue the extraction of the Root in Decimall parts: you shall set down after the separating Line, so many Circles as shall seeme requisite, marking them in like manner with Points above and beneath.

15. A Table shewing both the *Divisores*, and *Gnomons* for the Investigation of the single Sides in Adfected Æquations; collected and to be continued out of the latter Analyticall Table. And note that all the *Species* of every Coefficient are affirmative, if the Coefficient it selfe be adfirmative; and negative, if it be negative.

For

For the first Side	For the single Sides following, to make up the *Gnomon*.	
Aq BA	2AE . Eq BE .	} =Cq
Ac BAq CqA	3AqE . 3AEq . Ec B2AE . BEq . CqE .	} =Dc
Aqq BAc CqAq DcA	4 AcE . 6AqEq . 4AEc . Eqq B3AqE . B3AEq . BEc . Cq2AE . CqEq . DcE .	} =Fqq.
Aqc BAqq CqAc DcAq FqqA	5AqqE . 10AcEq . 10AqEc . 5AEqq . Eqc B4AcE . B6AqEq . B4AEc . BEqq . Cq3AqE . Cq3AE . CqEc . Dc2AE . DcEq . FqqE . &c.	} =Gqc

16. The Diviſors are every where taken out of thoſe things, which are given in meaſure, being diſpoſed in their juſt order, and put together, reſpect of their Signes being had.

17. If the higheſt *Poteſtas* of any Æquation bee Negative, that Æquation is ambiguous.

18. The Firſt ſingle Side ſhall bee extracted by theſe Rules, deſumed out of the two Conſectaries, Sect. 6. and 8.

Firſt, If the Coefficient ſet in its right place, ſtand ſo farre backward, that it ſcarce reacheth to the firſt Point of the *Poteſtas* Reſolvend; nor (though Analytically reduced) maketh therein any notable mutation: it may in the Extraction of the Firſt ſingle Side bee quite neglected.

Second. If the Coefficient ſet in his right place, ſtand forth very forward, and be adfirmative, you ſhall put it backe from point to point, untill there be place for Diviſion. By which Diviſion the Quotient found ſhall be referred to the degree of Adfection. Which is alſo to be underſtood in the Extraction of the leſſer Root of an ambiguous Equation.

Third. But if the Coefficient be Negative, and conſiſt of more Points, then the *Poteſtas* reſolvend; ſet the Deficient places bee ſupplied with Circles prefixed: and for the Firſt ſingle Side, take the Root of the Coefficient, according to the *kinde* thereof.

Fourth. If on both Sides the Points be equall, and the numbers in the firſt Point, both of the Coefficient

For the | For the ſingle Sides following,

ent, and of the *Poteſtas* Reſolvend, differ not much: the Coefficient being reduced by (Analyticall Multiplication) unto the Ranke of the *Poteſtas*, by its Root of that *kinde* for which it is Pointed, extracted under the congruent Point; if it be Negative, is to be added to the *Poteſtas* Reſolvend; but if Adfirmative, to be ſubſtracted. For if $Ac+CqA=Dc$, then $Ac=Dc+CqA$. But if the greater Side of an ambiguous Æquation be ſought, the *Poteſtas* Reſolvend is to be taken out of the Coefficient reduced. For if $CqA-Ac=Dc$, then it ſhall be $Ac=CqA-Dc$. Which done, the Root of the Summe or Difference ſhall be the Firſt Root to be extracted. And Note, that the greater Side of an ambiguous Æquation is found out ſometimes by Diviſion, ſometimes by extracting the Root out of the Coefficient; but moſt commonly by Reduction of the Coefficient.

19. And theſe Precepts being carefully obſerved, That, at length ſhall be the Firſt true ſingle Side, which firſt of all affordeth ſuch a Diagonall, together with the Coefficients multiplied, as the condition of the Æquation requires, according to the precedent Table; and all being gathered into one Summe (having diligent regard to the Signes and ſeats or places) ſhall bring forth a number not greater then the *Poteſtas* Reſolvend, from whence it is to be ſubducted. And it is to be Noted, that a Negative number how great ſoever, is leſſe then every both Adfirmative and leſſer Negative: As -4 is leſſe then 1, or then -1. Alſo that

Sub-

Subduction changeth the Signe of the number to bee ſubducted : As out of 4 take 6, Reſt 4−6, that is, −2. And out of −4 take −6, Reſt −4 + 6, that is, 2. Laſtly, out of 4 take −6, Reſt 4+6, that is, 10. Wherefore in the Extraction of the Firſt ſingle Side, ſome trialls muſt be made, untill you finde out the true Side; which by the next greater you ſhall moſt certainly know.

20. In the conſtitution of the Diviſor for the finding out of the Second Side; the Coefficient being drawne into any degree, the ſeat thereof muſt be ordered according to the Punctation of its proper degree : that is, the ſeat of the Coefficient under a Side, ſhall be diſtant One place toward the left hand, from the point or ſeat of the Coefficient it ſelfe : the ſeat of the Coefficient under a Quadrat, Two places: under a Cube, Three : &c. And to avoid confuſion, it will do well in the Remainder of the *Poteſtat* Reſolvend, to diſtinguiſh only thoſe Punctations, which ſerve for the Extraction of the preſent Root.

21. Then the Second ſingle Side ſhall bee educed thus : Let the Diviſors of every *kinde* found by the precedent Table, and diſpoſed in their juſt order, be gathered into one Summe; and let the Remainder of the *Poteſtas* Reſolvend be divided by that totall Diviſor. For the Quotient conſidered (if need require) according to the Lawes of Analyticall Diviſion, ſhall give the Second ſingle Side to bee extracted. But in this Inveſtigation many times, eſpecially

if the Aggregate of the negative dividing magnitudes be neare equall to the Aggregate of the Adfirmative (ſo that the Diviſor be much leſſe then the Remainder of the *Poteſtas* Reſolvend) there will fall out great uncertainty: which nevertheleſſe the witty Analyſt will eaſily avoid.

22. Let this therefore be a perpetuall Rule: That, at length is the Second true ſingle Side, which firſt of all affordeth a Gnomon; conſiſting of Complements of every *kinde*, and Coefficients multiplyed, as the condition of the Æquation requireth, according to the precedent Table; and all being gathered into one Summe, having diligent regard every where both to the Signes and Seats; which Gnomon is not greater then the *Poteſtas* Reſolvend, from whence it is to be ſubducted. Wherefore ſome triall muſt be made, untill you find the true Root; which alſo, by the next greater, you ſhall moſt certainly know.

23. All the other ſingle Sides after the Second, are moſt eaſily gotten only by ordinary Diviſion.

24. If the Adfections be compounded of Adfirmatives and Negatives: the foregoing Precepts are mixtly to be uſed with skill and judgement. And in eſtimating the Sides, a greater Adfection ſhall ever weigh downe a leſſer. But frequent exercitation both in the *Geneſis* and *Analyſis* of *Poteſtates* of every Kinde, will render this whole Analyticall buſineſſe eaſie enough and familiar; which to declare in words would prove moſt difficult.

25. But

25. But because I have formerly sometimes mentioned a necessity of trialls; which indeed in manifold Adfections, and where the degrees are higher, would be exceeding troublesome. I shall here in manner of a *Coronis* set downe two wayes of easing that triall. One by *Depression*, out of Chap: XVI. Sect: 7. *Clavis*. Another by the *Canon* of 10000 Logarithmes. But in both these wayes, if the Æquation be ambiguous, all the Signes thereof are to be changed.

26. To finde the single Sides by Depression.

If the First single Side be sought. In every severall given *Species* of the Æquation, cut off with a separating line, all the Points after the First. Then divide all the *Species* by the *Latus*; that is, Depresse them one degree.

Exampl: I. $1qq - 72c + 238600l = 8725815$. This by Depression is made $1c + 238,\underline{6} - 7,\underline{2q} = L)\ 872,\underline{5}$

If A be 4. Then $4)\ 872,\underline{5}\ (218,\underline{1}$, just.

And $+64 + 1238,\underline{6} - 115,\underline{2} = 187,\underline{4}$, lesse then just.

If A be 5 Then $5)\ 872,\underline{5}\ (174,\underline{5}$, just.

And $+125 + 238,\underline{6} - 180,\underline{0} = 183,\underline{6}$; greater then just.

Therefore the true Side $A = 5 - 1$, that is, 4.

Exampl: II. Of an ambiguous Æquation.

1c-3257l = -45744. This by Depression is made

1q-325=L)-457.

If A be 4. Then it shall be 4)—457 (—114, just.

And + 16—3215 = -1615, lesse then just.

If A be 5, Then 5)—457 (—91, just.

And + 25—325 = —715, greater then just.

Therefore the true Side A = 5—1, that is, 4.

If the Second single Side be sought: In every Se- verall *Species*, cut off all the Points after the Second. Then divide all by the Quadrat, that is, Depresse them two Degrees. As in Example I.

1qq—72c + 238600l = 8725815. This De- presse becommeth 1q+L) 238600-721 = Q 8725815.

If A be 47: then 2209) 8725815 (3949. Just.

And 2209+5077—3384 = 3896. Lesser then just.

If A be 48: then 2304) 8725815 (3787. Just.

And 2304+4971-3456 = 3819: greater then just.

Therefore the true Side is, 48—1, that is, 47.

27. To find the Second single Side by Logarithme.

The *Index* of every Logarithme is taken out of the Table in the beginning of the *Clavis*, according to the distance of its First figure before or after the Unit place, the *Index* whereof is 0. The same figures therefore disposed in the same order, have the same

Log

Logarithme: But the *Indices* may be diverſe. As of the number 436, the Logarithme is 2,6394865. But of the number 43600, the Log: is 4,6394865. And of the number 4,36, the Log: is 0,6394865. Laſtly, of the number 0,00436, the Log: is $\bar{3}$,6394865.

The Summe of two Logarithmes, is the Logarithme of the Product of their Valors: and the difference is the Logarithme of the Quotient. As becauſe 4,36×9=39,24: the Logarithme of it is 1,5937290 =0,6394865 + 0,9542425: And becauſe 9) 39,24 (4,36: the Logarithme hereof 0,6394865 = 1,5937290—0,9542425.

The Logarithme of the Side, drawne into the *Index* or number of Dimenſions of any *Poteſtas*, is the Logarithme of the ſame *Poteſtas*. As becauſe, of the number 436, the Log: is 2,6394865: It ſhall be that 2,6394865×2=Log: of Q:436. And 2,6394865×3 =Log: of C: 436: And 2,6394865×4=Log: QQ: 436. &c.

The Logarithme of any *Poteſtas*, divided by the number of its Dimenſions, ſheweth the Logarithme of its Root.

If in a continued *Series* of Geometricall Proportionalls, the Logarithme of the Firſt Terme, be taken out of the Logarithme of the Second; the Remainder ſhall be the Logarithme of the Ration. Which, if it be drawn into the number of all the Termes, wanting one

which is the number of the Rations;) and then be increased with the Logarithme of the first Terme; shall be the Logarithme of the last Terme.

28. And let this suffice concerning the knowledge of Logarithmes : which being understood, the following Examples diligently observed, will make the rest of the Worke easie. In which also all the Punctations, after the two First, are to be cut off with a separating Line.

Exampl: I. $1qq - \dot{7}2\dot{c} + 238\dot{6}00\dot{l} = 87\dot{2}5815\dot{.}$ Just.

Let the two First single Sides be,

		-72	$+238600$
47.	1,67209 | 8	1,85733	5,37767
Cu:	5,01629	5,01629	1,67210
QQ:	6,68839	6,87362	7,04977
	+4880-	−7475	+11213

And +4880+11213−7475 = +8618: lesse then just.

48.	1,68124 | 1	1,85733	5,37767
Cu.	5,04372	5,04372	1,68124
QQ.	6,72496	6,90105	7,05891
	+5308	−7963	+ 11455

And +5308+11455−7963 = +8800 : greater the just.

Therefore the true Root shall be 48−1, that is. 47

Examp

Eampl:II. 1c—32571 =—45744. Juſt.
Let the two Firſt ſingle Sides be,

		1—32571
48.	1,681124 | 1	3,51282
Cu.	5,04372 |	1,68124
	+1106	5,19406
		—1563

+1106—1563 =—457, leſſe then juſt.

49.	1,690196	3,51282
Cu.	5,07059 |	1,69020
	+1176	5,20302
		—1596

+1176—1596 =—420; greater then juſt.

Therefore the true Root ſhall be 49—1, that is, 48

The Second Side may be ſought by the Logarithmes, Depreſſion alſo preceding. As in Example V,

1qq —1246|00q = 089726256 : which Depreſſion Quadraticall is made 1q —1246 = Q.) 89726.

Suppoſe the two Firſt ſingle Sides,

		89726|1
34.	1,53148	3,95337
Q.	3,06296	3,06296
	+ 1156	0,89041: valor 7,77 Juſt.

+1156—1246 =—90: leſſer then juſt.

36. 1,55630 | 3,95337
3,11260 | 3,11260

† 1296 0,84077: valor 693 Just.

† 1296—1246=+50: greater then just.

The true Root therefore falleth betweene 34. and 36.

Thus have I in XXVIII Sections or Precepts (which is a perfect number) finished the doctrine concerning the Resolution of Adfected Æquations in numbers, by the adsistance of God the giver of all good things. His therefore be all the praise, honour, and glory, for ever. *Amen.*

Some

Some Examples of Æquations reſolved in numbers.

I Forbear in theſe Examples to touch the Analyſis of Quadratic Æquations, and of all other, which conſiſt of three *Species* equally aſcending in the order of the Scale: Becauſe in Chap. XVI, Sect. 9, *Clavis*, a more eaſie way is taught, then can be performed in this generall method: And here I proceed to the Examples of Æquations otherwiſe affected.

Laſtly, in the end I ſet downe ſome Notes upon the Examples, in which the reaſon of the worke in the inveſtigation of the ſingle Sides, is ſhewed out of the former Precepts.

I will begin with the Reſolution of that Æquation, the Conſtruction whereof was ſhewed in the very firſt entrance of this Treatiſe, Namely:

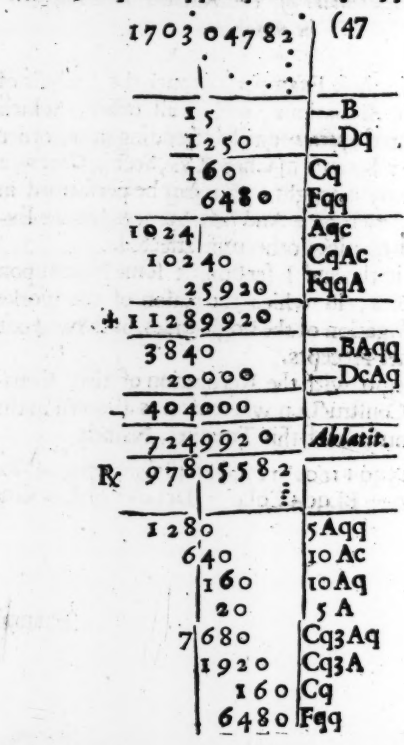
$$1qc - 15qq + 160c - 1250q + 6480l = 17030478^2.$$

That is, $Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$

Example

Example I.

1qc−15qq+160c−1250q+06480l = 17030478 2.
That is, Lqc−BLqq+CqLc−DcLq+FqqL = Gqc.



+142	50040	
38	40	- B4Ac
1	440	- B6Aq
	240	- B4A
	15	- B
1	0000	- Dc2A
	1250	- Dc
-40	87665	
+101	62375	Diviſor.
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	- B4AcE
70	560	- B6AqEq
8	2320	- B4AEc
	36015	- BEqq
7	0000	- Dc2AE
	61250	- DcEq
-355	56465	
+978	05582	*Ablatit.*

Exam.

1c + 420000l = 24765171 3.

That is, Lc + CqL = Dc.

247	65	17	13	(417			
42	0	0	0	Cq			
64				Ac			
168	0	0	0	CqA			
232	0	0	0	*Ablatit.*			
Rc 15	65	17	13				
48				3Aq			
12				3A			
42	0	0	0	Cq			
9	12	0	0	*Diviſor.*			
48				3AqE			
12				3AEq			
1				Ec			
42	0	0	0	CqE			
9	12	1	00	*Ablatit.*			
Rc 6	53	07	13				
50	43			3Aq	4	1	
1	23			3A	16		
42	0	0	0	Cq		8	
92	55	3	0	*Diviſor.*			1
35	30	1		3AqE	16	8	1
6	02	7		3AEq			
	3	43		Ec			
29	40	0	0	CqE			
6	53	0	713	*Ablatit.*			

Exam

Example III.

$1c + 1007q = 24761936$,
That is, $Lc + BLq = Dc$.

2	4	7	6	1	9	3	6	(417	
	1	0	0	7				B	
	6	4						Ac	
1	6	1	1	2				BAq	
2	2	5	1	2				*Ablatit.*	
Rc	2	2	4	9	7	9	3	6	
		4	8					3Aq	
			1	2				3A	
		8	0	5	6			B2A	
			1	0	0	7		B	
	1	3	0	7	6	7		*Diviſor.*	
		4	8					3AqE	
			1	2				3AEq	
					1			Ec	
		8	0	5	6			B2AE	
			1	0	0	7		BEq	
	1	3	0	7	7	7		*Ablatit.*	

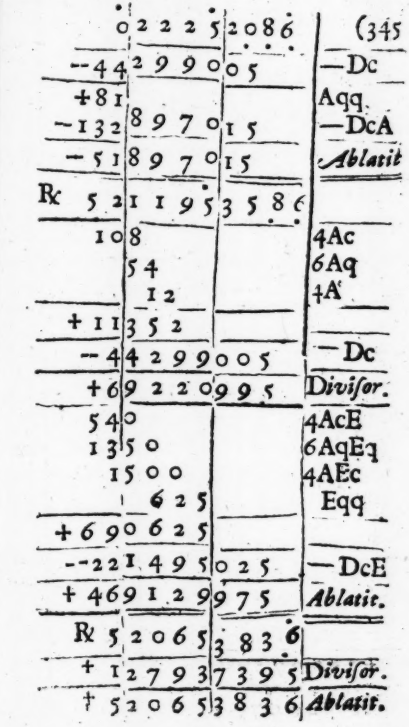
Rc									
	9	4	2	0	2	3	6		
		5	0	4	3			3Aq	
				1	2	3		3A	
		8	2	5	7	4		2BA	
				1	0	0	7	B	
	1	3	3	2	2	7	7	Diviſor.	
	3	5	3	0	1			3AqE	
		6	0	2	7			3AEq	
				3	4	3		Ec	
	5	7	8	0	1	8		B2AE	
		4	9	3	4	3		BEq	
	6	9	4	0	2	3	6	*Ablatit.*	

4	1
16	8
	1
16	81

Example

1qq -- 44299005 l = 2225208 6

qq — DcL = Fqq.



0	2225	2086	(345
-- 44	2990	05	— Dc
+ 81			Aqq.
- 132	8970	15	— DcA
- 51	8970	15	*Ablatit*
R 52	1195	3586	
108			4Ac
	54		6Aq
	12		4A
+ 11	352		
-- 4	4299	005	— Dc
+ 6	9220	995	*Divisor.*
540			4AcE
1350			6AqEq
1500			4AEc
	625		Eqq
+ 690	625		
-- 22	1495	025	— DcE
+ 46	9129	975	*Ablatit.*
R 5	2065	3836	
+ 1	2793	7395	*Divisor.*
+ 5	2065	3836	*Ablatit.*

1qq - 1246qq = 08972625 6, Lqq - CqLq = Fqq.

	08972	6256	(354
-	124600		- Cq
+	81		Aqq
-	112	1400	- CqAq
-	311400		*Ablatit*
Rr	32	037262	56
	108		4Ac
	54		6Aq
	12		4A
+	11	352	
	7	476000	- Cq2A
		124600	- Cq
-	7	600600	
+	3751400		*Divisor*
	540		4AcE
	1350		6AqEq
	1500		4AEc
		625	Eqq
+	690625		
	3738000		- Cq2AE
	3150000		- CqEq
-	4048500		
+	2856750	0	*Ablatit*
Rr	34	697625	6
		8489	1800 *Divisor*
†	34	697625	6 *Ablatit*

Exam

Example VI.

$1qq - 340c = 62106096$
$Lqq - BLc = Fqq.$

6	2 1 0 6 6 0 9 6	(354
−3	4 0	−B
+81		Aqq
−−91	8 0	BAc
−1	0 8 0	*Ablatii*
Rx 17	0 1 0 6 6 0 9 6	
10	8	4Ac
	5 4	6Aq
	1 2	4A
+11	3 5 2	
9	1 8 0	−B3Aq
	3 0 6 0	−B3A
	3 4 0	−B
−−9	4 8 9 4 0	
+ 1	8 6 2 6 0	*Divisor*
5	4 0	4AcE
1 3	5 0	6AqEq
1	5 0 0	4AEc
	6 2 5	Eqq

+69	06	25		
45	90	0		-B3AqE
7	65	00		-B3AEq
	42	50	0	-BEc
−53	97	50	0	
+15	08	75	0	*Ablatit*
Rc 1	92	31	60	96
+	46	92	90	60 *Divisor*
+1	92	31	60	96 *Ablatit*

Exam. VII.

Example VII.

1qq — 771080001 = 085530576, Lqq-DcL = Fqq.

085530576	(426	
−771080000	—Dc	
+256	Aqq	
−308432000	—DcA	
−52432000	*Ablatie*	
Rc 53287305 76		4 . 2
256	4Ac	16
96	6Aq	16
16	4A	4
+26576		1764
—771080000	—Dc	
+18865 2000	*Divisor*	
512	4AcE	
384	6AqEq	4 . . 2
128	4AEc	646 8
16	Eqq	948
+551696		74088
—154216000	—DcE	
+39748000	*Ablatit.*	
Rc 135393076		
+2203040 80	*Divisor*	
+135393076	*Ablatit.*	

Example VIII.

$32001 - 1c = 46577$ An ambiguous
$CqL - Lc = Dc$ Æquation.

4 6 5 7 7		(47 Greater Root.
3 2 0 0	Cq	
--64	-Ac	
+12800	CqA	
+6400	*Ablatit.*	
Rc-17423		
48	-3Aq	
12	-3A	
-492		
+3200	Cq	
--1720	*Divisor.*	
336	-3AqE	
588	-3AEq	
343	-Ec	
-39823		
+22400	CqE	
-17423	*Ablatit.*	
Rc 0 0 0 0		

Exam. IX

$$32001 - 1c = 46577$$

46	577		(157 Leſſer Root.
32	00	Cq	
--	1	Ac	
+32	00	CqA	
31	00	*Ablatit.*	

Rc 15	577		
	3	3Aq	
	3	3A	
-	33		
+32	00	Cq	
28	70	*Diviſor.*	
1	5	--3AqE	
	75	-3AEq	
	125	--Ec	
-2	375		
+16	000	CqE	
13	625	*Ablatit.*	

Rc 1	9522	000	
	252	05	*Diviſor.*
	745	107	*Ablatit*

Rc 2068 9300 &c.

$53q - 1c = 13254$ An ambiguous.
$BLq - Lc = Dc$ Æquation.

13	2 5 4	(47	Greater Root.
5	3	B	
−64		-Ac	
+84	8	BAq	
+20	8	*Ablatit*	
Rx −7	5 4 6		
4	8	-3Aq	
	12	-3A	
−4	92		
4	24	B2A	
	53	B	
+4	293		
−	627	*Diviſor*	
33	6	-3AqE	
5	88	-3AEq	
	343	-Ec	
−39	823		
29	68	B2AE	
2	595	BEq	
+32	2 77		
−7	546	*Ablatit*	
Rx 0	0 0 0		

Example XI.

$$53q - 1c = 13254$$

13254	(20105 Leſſer Root.	
53	B	
−8	-Ac	
213	BAq	
132	*Ablatit*	
Rx	5400000	
	−1200000	−3Aq
	−600	−3A
	−1200600	
	2120053	B2A
	53	B
	+2120553	
	919930	*Diviſor*
	−6000000	−3AqE
	−15000	−3AEq
	−125	−Ec
	−6015015125	
	10600000	B2AE
	13325	BEq
	+10613325	
	4598237 5	*Ablatit*
Rx	90176250000	&c

Example

Example XII.

$$6003\dot{4}l - 1c = \dot{1}02\dot{3}\dot{7}6\dot{8}$$

$$CqL - Lc = Dc.$$

	$\dot{1}$	0 2 $\dot{3}$	7 6 $\dot{8}$	(236 Greater Root.
	6	0 0 3	4	Cq
—	8			-Ac
+	1 2	0 0 6	8	CqA
+	4	0 0 6	8	*Ablatit.*
R-	2	9 8 $\dot{3}$	0 3 2	
	1	2		-3Aq
		6		-3A
—	1	2 6		
+		6 0 0	3 4	Cq
—		6 5 9	6 6	*Divisor.*
	3	6		-3AqE
		5 4		-3AEq
		2 7		-Ec
—	4	1 6 7		
+	1	8 0 1	0 2	CqE
—	2	3 6 5	9 8	*Ablatit.*
R-		6 1 7	0 5 $\dot{2}$	
		-9 6	3 6 6	*Divisor.*
		6 1 7	0 5 2	*Ablatit.*

Exam. XIII.

Example XIII.

$$600341 - 1c = 1023768$$

1	0	2	3	7	6	8	17135 Leſſer Root.
	6	0	0	3	4		Cq
		—	1				-Ac
+	6	0	0	3	4		CqA
+	5	9	9	3	4		*Ablatit*

Rc	4	2	4	4	2	8	
				3			-3Aq
					3		-3A
			—	3	3		
†	6	0	0	3	4		Cq
	5	9	7	0	4		*Diviſor*
			2	1			-3AqE
			1	4	7		-3AEq
				3	4	3	-Ec
		—	3	9	1	3	
†	4	2	0	2	3	8	CqE
+	4	1	6	3	2	5	*Ablatit.*

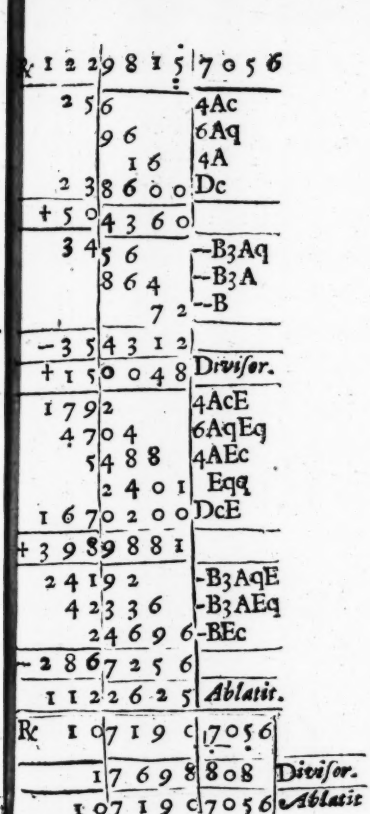
Rc	†	8	1	0	3	0	0 0
		5	9	1	6	1	9 *Diviſor.*
		6	0	1	6	1	8 9 *Ablatit.*

Rc	2	0	8	6	8	1	1	0 0 0
		5	9	1	5	6	2	5 7 Divisor
	1	7	7	5	5	5	6	9 0 3 Ablatit
Rc	3	1	1	2	5	4	0	9 7 0 0 0
		5	9	1	5	3	6	3 7 9 1 Divisor.
	2	9	5	7	6	7	1	6 1 6 2 5 Ablatit
Rc	1	5	4	8	6	9	3	6 3 7 5 0 0 0 &c.

Example XIV.

$$1qq - 72c + 2386001 = 87258151756.$$
$$Lqq - BLc + DcL = Fqq.$$

8	7	2	5	8	1	5	7	0 5 6 (476
		−	7	2				-B
+	2	3	8	6	0	0		Dc
		2	5	6				Aqq
		9	5	4	4	0	0	DcA
+	1	2	1	0	4	0	0	
−		4	0	6	8			BAc
+	7	4	9	6	0	0		Ablatit.


Rc 1 2 2 9 8 1 5 7 0 5 6
2 5 6 4Ac
9 6 6Aq
1 6 4A
2 3 8 6 0 0 Dc
+ 5 0 4 3 6 0
3 4 5 6 -B3Aq
8 6 4 -B3A
7 2 -B
- 3 5 4 3 1 2
+ 1 5 0 0 4 8 Divisor.
1 7 9 2 4AcE
4 7 0 4 6AqEq
5 4 8 8 4AEc
2 4 0 1 Eqq
1 6 7 0 2 0 0 DcE
+ 3 9 8 9 8 8 1
2 4 1 9 2 -B3AqE
4 2 3 3 6 -B3AEq
2 4 6 9 6 -BEc
- 2 8 6 7 2 5 6
1 1 2 2 6 2 5 Ablatit.
Rc 1 0 7 1 9 c 1 7 0 5 6
1 7 6 9 8 8 0 8 Divisor.
1 0 7 1 9 c 7 0 5 6 Ablatit

Exem. XV.

31-1c = 1 2 5 8 6 4 0 7 8 2 1 0 0 CqL-Lc = D

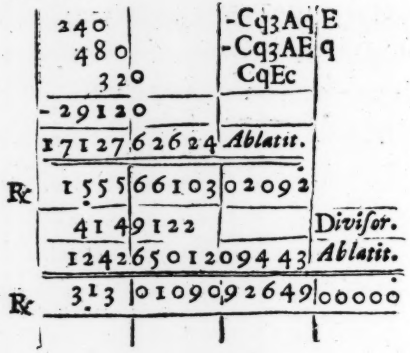
	1 2 5 8 6 4 0 7 8 2 1			(0,4499
	3 3	Cq		Subtense
	- 6 4	-Ac		Gr: 26.
+ 1 2		CqA		
	1 1 3 6	*Ablatit.*		
Rc	1 2 2 6 4 0			
	4 8		-3Aq	
	1 2		-3A	
	-4 9 2			
	+ 3		Cq	
	2 5 8 0		*Divisor*	
	1 9 2		-3Aq	E
	1 9 2		-3AEq	
	6 4		-Ec	
	- 2 1 1 8 4			
+	1 2		CqE	
	9 8 8 1 6	*Ablatit*		
Rc	2 3 8 2 4 7 8 2			
	2 4 1 7 8 8	*Divisor.*		
	2 1 6 6 5 1 5 1	*Ablatit.*		
Rc	2 1 5 9 6 3 1 1 0 0			
	2 3 9 5 0 6 2 3	*Divisor.*		
	2 1 5 4 5 8 5 5 0 1	*Ablatit.*		
Rc	0 5 0 4 5 5 9 9 0 0 0			

1qc-5c+5l=1|147 15287 27020 92

Lqc-CqLc+FqqL=Gqc.

1	147 15	2872 7	02092
+5		Fqq	
	—5	—Cq	
	32	Aqc	
10		FqqA	
+10003 2			
	—40	CqAc	
	96032	*Ablatit.*	
R	18683	2872 7	
	80		5Aqq
	80		10Ac
	40		10Aq
	10		5A
5			Fqq
+500884 10			
60			-Cq3Aq
30			-Cq3A
	5		-Cq
—6305			
+4378341 0			*Divisor.*
320			5AqE
480			10AEq
2560			10AqEc
	2560		5AEqq
	1024		Eqc
20			FqqE
+	20039	62624	

(02437 Subtense *Gr:* 14.



	240		-Cq3Aq	E
	480		-Cq3AE	q
	320		CqEc	
	-29120			
	17127	62624	*Ablatit.*	
℞	1555	66103	02092	
	4149122			*Divisor.*
	1242	65012	09443	*Ablatit.*
℞	313	01090	92649	00000

Notes

Notes upon the former Examples.

IN the Examples of the Sections 26 and 28, I call that number Juſt, which ariſeth from the application of the *Poteſtas* Reſolvend to the degree of the ſuppoſed Side, by which the depreſſion is made. For this is the meaſure, unto which the other *ſpecies* being lawfully aggregated, ought to be equall. As in Exampl: 1. of Section 26, 1c+238,6-7,2q=L) 872,5. If for the firſt ſide be ſuppoſed 5: Ought to bee C:5: +238,6-7,2Q:5: = 872,5 divided by 5: that is, 125+238,6-(7,2×25) 180, namely, 183,6 æquall to 174,5 Juſt. But it is greater, and therefore the true ſide is leſſer then 5. Suppoſe it againe therefore 4: and make triall whether C:4:+238,6-7,2 Q:4: be equall to 872,5 divided by 4.

But leſt that in theſe Examples, as alſo in thoſe that follow, theſe trialls be undertaken meerly by chance; Let theſe Monits be given:

Firſt, If the homogene *Poteſtas* of the ſide extracted exceed the *Poteſtas* Reſolvend: or If the magnitudes augmenting the *Poteſtas* Reſolvend, exceed thoſe that imminiſh it: The true ſide A ſhall bee (ever for the moſt part) leſſe than the ſide extracted: But if otherwiſe, greater. As in this Æquation,

1ct 2600001 = 1809317,13.

Secondly, If the Divisores under the same signe with the Remainder of the *Porisma* Resolvend, exceed those which are under the contrary signe: The true side E shall be (ever for the most part) lesse than the Quotient: But if otherwise, greater: As in this Æquation, $67681 - 1c = 214278$. The solution of these three Æquations in practice, shall be shewed after the Notes.

Thirdly, If after these Monits, notwithstanding there remaine any doubt; triall shall be most fitly begunne at 5: And from thence the inquisition continued by odde numbers; whether it be done by Depression, or by the Logarithmes.

These things being premonished, it remaines that we examine the Examples themselves.

For Exampl: I. $\sqrt{qc1703}$ est 44, by Sect: 18 Rule 1. For, as appeares from Sect: 7, in the first point there is no great mutation made by the Coefficients Analytically reduced. Wherefore the true side A shall be 4.

The true side E is lesse then the Quotient 9: because the Divisores under the signe + (which is the signe of R) exceed those that are under the signe −.

For Exampl: II. 42) 247 (8-, by Sect: 18, Rule 3. For 42 reduced Analytically, by Sect: 7 and 8, is made 252: greater then 247. And the true side A lesse then 6, because $C.6-$ exceedeth 247, 6.

For Exampl: III. 16) 247 (24+Q.5-: by Sect: 18 Rule 2.

F

For Exampl: IV. √c 44,3 is 3 +, by Sect : 18, Rule 3. Wherefore the true side A is 3.

The true side E is lesse then the Quotient 8-, by Monit 2.

For Exampl: V. √q 1214 is 3 +, by Sect: 18, Rule 3. Wherefore the true side A is 3.

The true side E is lesse then the Quotient 9-, by Monit 2.

For Exampl: VI: the laterall coefficient 3,4 multiplied Quadrato-quadraticly, and augmented with 612, is made 140, QQ:3 + : by Sect : 18, Rule 4. Wherefore the true side A is 3.

The true side E is lesse then the Quotient 9-, by Monit 2.

For Exampl VII. √c 77 is 4, by Sect: 18, Rule 3. Wherefore the true side A is 4.

For Exampl: VIII. √q 32 is 5,65, into 32 is made 180,8, mi 4615, remains 144, C:5: by Sect: 18, Rule 4. But 144 exceedeth 4615. Wherefore the true side A is lesse then 5, by Monit 1.

The true side E is lesse then the Quotient 10, by Monit 2.

For Exampl: IX. XI. XIII. The Solution is very easie by Division, according to Sect: 18, Rule 3.

For Exampl: X. C: 5: ct 125, mi 13, remaines 112, C:5-: by Sect: 18, Rule 4. But 112 exceedeth 13. Wherefore the true side A is lesse then 5, by Monit 1.

The true ſide E is leſſe then the Quotient 12-, by Monit 2.

For Exampl: XII. √96 is 24, into 6 is made 12, mi 1, remaines 11, C:24, by Sect: 18, Rule 4. But 11 exceedeth 1. Wherefore the true ſide A is a little leſſe then 24, by Monit 1.

The true ſide E is leſſe then the Quotient 5-, by Monit 2.

For Exampl: XIV. QQ:7|2: is —2687. And √c 238|6 is 6|2 whoſe QQ is +1480. Then —2687+1480=-1207 : This added to 872, giveth 2079, QQ:64: by Sect: 18, Rule 4, And becauſe the adjectitious--2687 is greater then the ablatitious +1480, the true ſide A ſhall bee leſſe then 6, by Monit 1.

The true ſide E is leſſe then the Quotient 9, by Monit 2.

For Exampl: XV, XVI. Becauſe in both the leſſe Root of an ambiguous Æquation is ſought, nor do the Coefficients, though reduced, withſtand, the Analyſis ſhall bee made by Diviſion, according to Sect: 18, Rule 1.

The practice of the Example in the firſt Monit.

1c+26,0000=1809 3171 3.

1809 (4, the ſide A

26,0 Cq.

√q 26 est 5, into 26 is made 130, which taken out of 180, there remaines 50, C: 3 +: which is lesse then 180. Wherefore the true side A is greater then 3.

The practice of the former Example in the second Monit.

$$1\dot{5}68\dot{1} - 1c = 2\dot{1}9\dot{5}\dot{2}$$

$2\dot{1}$	$9\dot{5}\dot{2}$	(28, the two first sides.
15	68	Cq
−8		Ac
+31	63	CqA
+23	36	*Ablat*
℞ −1	$40\dot{8}$	
1	2	−3Aq
	6	−3A
−1	26	
+1	568	Cq
	308	*Divisor*

The signe of ℞ is −. But −1|26 is lesse then +1|568. Wherefore the true side E is greater then the Quotient 4.

The practice of the latter Example in the second Monit.

$$67681 - 1C = 214273.$$

21	42	73	(47, the two first sides.
6	7	68	Cq
−64			-Ac
+270	72		CqA
+2067	2		*Ablat*
R+7	553		
	48		-3Aq
	12		-3A
−4	92		
+67	68		Cq
+18	48		*Divisor.*

The signe of R is +. But the Divisor collected of the degrees of A negative, is lesse then the coefficient divisor adfirmative; that is −4,92 is lesse then +67,68. Wherefore the true side E shall be greater then the Quotient 4.

In Sect: 28. I have in a few words delivered the doctrine of the Logarithmes; But cleerly enough, especially for the three former *Species* of Numeration, to wit, Addition, Subduction, and Multiplication.

The

The Operation in Adding, and Subtracting, if the Indices bee adfirmative, differs nothing from the common way of Integres: and but little alſo, if they be negative, as appeareth by theſe Examples:

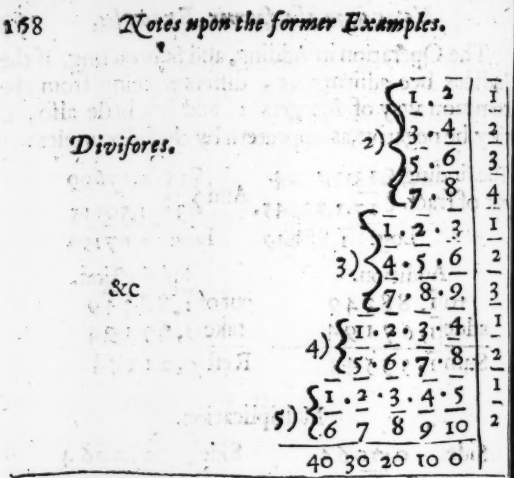
The finding out of fract.	13.1,1394.	And	15. 1,17609
	17.1,23045.		32. 1,50515
	Log: $\bar{1}$,88349		Log: $\bar{1}$,67194

Addition.		Subduction.	
to	$\bar{1}$, 88349	out of	$\bar{1}$, 88349
adde	1, 67194	take	1, 67194
Sum	$\bar{1}$, 55543	Reſt	0, 21155

Multiplication.

Side	0,00064	Side	0,00064
	3 * $\bar{3}$, 80614		2 * $\bar{3}$, 80614
Cube	$\bar{7}$, 41842	Quadr.	$\bar{5}$, 61228

The difficulty of dividing a Logarithme, that hath a negative Index, by 3, 5, 4, 5, &c. conſiſteth in the inveſtigation of the Index of the Quotient. For which purpoſe this Table ſerveth.



In this Table the Divisores ſtand towards the left and within the crooked line.

Then, towards the right hand, follow the negative Indices of the Logarithmes to be divided.

By theſe in the ſeverall rowes ſtand the negative of the Quotients.

The numbers which are written underneath 0,10,20,30,40, ſhew the numbers to be added to the firſt figure of the Logarithme of the dividend whoſe negative Index is found above in the ſame columne, by the Diviſor. As if the Logarithme $\bar{7},4184$ be required to be divided by 3 : Seeke $\bar{7}$ by 3) and the collaterall $\bar{3}$ ſhall be given for the Index of the Quo-

Quotient: And the number 20 underneath; which added to 4, the firſt figure of the dividuall, maketh it 24: in which the Diviſor 3 is contained eight times.

Diviſion.

3) $\bar{7},41842$	2) $\bar{5},61228$
Side, $\bar{3},80614$	Side, $\bar{3},80614$

Of Anatociſme, or compound Uſury.

That is, the Analyticall Inveſtigation of ſix fundamentall Theoremes, by which all queſtions concerning Anatociſme, may very eaſily bee reſolved.

And it is to be noted, that an Engliſh Shilling containeth 12 pence: And a Pound containeth 20 Shillings: or 240 pence.

1. THE *Ratio* of the increaſe or gaine is firſt to be reduced to an equall *Ratio*, whoſe antecedent is 100, or 1. As if the *Ratio* be of Pence $19\frac{2}{5}$, or of ſhil. $1\frac{6}{5}$, for 1pound: Say, 240, $259\frac{2}{5}$; or 20 $21\frac{6}{5}$:: 100. 108::1. $1\frac{08}{100}$: to wit $\alpha.\beta.$ wherefore β is made of the common ſtocke α, in one whole yeare.

2. But if the payment be halfe yearly or quarterly, that is by $182\frac{5}{10}$ dayes, or by $91\frac{25}{100}$ dayes: For β, let

let the Logarithme of the yearely increase be multiplied by $\frac{1}{2}$ or by $\frac{1}{4}$: or rather by $\frac{182,5}{365}$ or by $\frac{91,25}{365}$ for the $\frac{1}{2}$ or $\frac{1}{4}$ of the yearely increase or gaine is commonly mistaken.

3. Because in Progression, the number of the *Ratio's* is lesse by unity, then N the number of the Tearmes, or Payments; the number of *Ratio's* shall be N-1. Also the Logarithme β drawne into N-1, shall be the Logarithme ω of the last tearme. Lastly, the Logarithme β drawne into N, shall be the Logarithme $\beta\omega$, that is, of the said β multiplied continually in it selfe, for the number of the Payments.

4. Wherefore $\beta\omega$ is made of α the stocke, or 1^{lb}, paid for N returnes. Hence ariseth two Theoremes.

Theo: I. $1^{lb}.\ \beta\omega :: Q^{lb}.\ Q^{lb}$ with the gaine in N returnes.

Theo: II. $\beta\omega.\ 1^{lb} :: Q^{lb}$ after N returnes. The present price.

5. Further because $\frac{\beta\omega-\alpha\beta}{\beta-\alpha}$, that is $\frac{\beta\omega-1}{\beta-1} = Z$, the summe of all the tearmes of the Progression (the last whereof is ω) and is therefore gotten from the Pension 1^{lb} intermitted for N returnes; Hence arise other two Theoremes.

Theo: III. $\beta-1.\ \beta\omega-1 :: Q^{lb}$ the Pension intermitted for N returnes. Pensions with the use to bee paid in the end.

Theo:

Theo: IV. $\beta\omega-1 . \beta-1 :: Q^{lb}$ to come. Pension equivalent to be paid in N returnes.

6. Lastly because $\beta\omega$ comes of 1^{lb} paid for N returnes: and $\frac{\beta\omega-1}{\beta-1}$ comes of the Pension 1^{lb} intermitted for N returnes; which in ready monies is equivalent to the price of the Pension: Say, $\beta\omega . 1^{lb} :: \frac{\beta\omega-1}{\beta-1}, \frac{\beta\omega-1}{\beta-1 \text{ in } \beta\omega}$: From whence therefore in N returnes shall come forth $\frac{\beta\omega-1}{\beta-1}$, the price of the Pension: Hence also ariseth two Theorems.

Theo: V. $\beta-1$ in $\beta\omega . \beta\omega-1 :: Q^{lb}$ Pension for N returnes. The price thereof in ready monies.

Theo: VI. $\beta\omega-1 . \beta-1$ in $\beta\omega :: Q^{lb}$ present. The Pension to be bought for N returnes.

Note that Q^{lb} signifieth any number of Pounds.

Example of a Pension during 10 yeares the Payments halfe yearely; in Proportion of 1 to 1,08: And N is 20: And the Logar: of 1,08 is 0,03342.

0,03342 in $\frac{1}{2}$

0,01671 Log: $\beta = 1,0396$

20 N

0,33420 Log: $\beta\omega = 2,159$

$\bar{2}$,59769 Log: $\beta-1 = 0,0396$

$\bar{2}$,93190

2,93190 Log: $\beta-1$ in $\beta\omega$

0,06408 Log: $\beta\omega-1=1,159$

Therefore,

0,06408

2,93190

1,13218 is the Log: of the price 13,56^{lb} for the Pension of 1^{l}.

2,93190

0,06408

2,86712 is the Logar: of the Pension 0,07376^{lb} for the price of 1^{l}.

To these found Log^{s}: adde the Log: of Q^{lb}.
Or multiply these values found by Q^{lb}.

The rule of falſe Poſition.

MUltiply the Poſitions by their alterne Errors. And if the Errors be of the ſame kinde, that is both exceeding, or both deficient; divide the difference of the Product, by the difference of the Errors; but if they be of divers kindes, divide the ſumme of the Products, by the ſumme of the Errors: And the Quotient ſhall give the number ſought.

For Demonſtration.

What number is that, which being drawne into B, doth produce the Plaine BA, to wit, BA pl.

Let it bee A-C	Let it bee A-D.
into B. BA-BC	into B. BA-BD

The errors therefore are,

BA pl-BA+BC. BA pl-BA+BD

Becauſe on both ſides the ſignes are alike; that thoſe which are equall may be cleared; It is needfull that Subſtraction be made, changing all the ſignes of the leſſer. For ſo thoſe which are equall mutually cancelling themſelves, the difference of the errors ſhall remaine, BC-BD.

BC defect.	BD defect.
A-D	A-C
BCA-BCD	BDA-BDC

And here alſo thoſe which are equall being expunged by Subſtraction; the Reduction comes to bee

BCA

BCA-BDA: which is the very difference of the Errors drawne into A.

Wherefore $\frac{BCA-BDA}{BC-BD} = A.$

Again, Let it be A+C. Let it be A-D
into B. BA+BC into B. BA-BD

The Errors therefore are,
BA+BC-BA pl. BA pl-BA+BD.

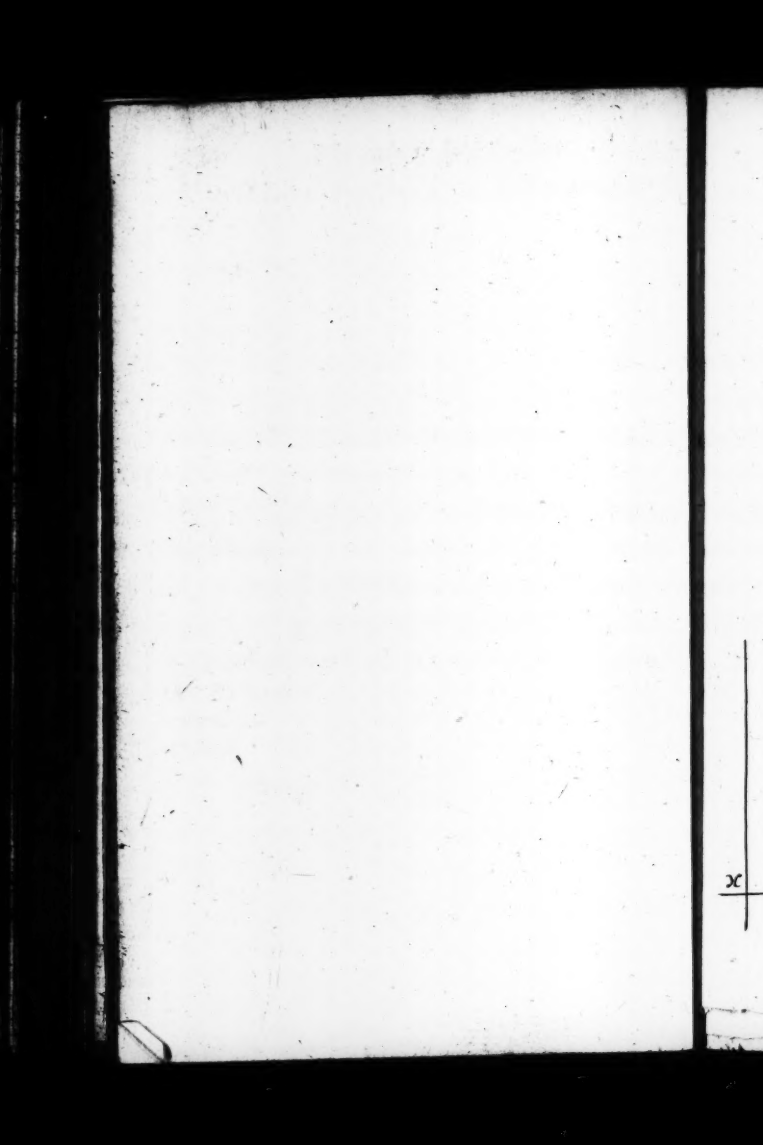
Becauſe on both ſides the ſignes are contrary, thoſe which are equall by Addition will cleare one another mutually, without the changing of any ſignes: And ſo the ſumme of the Errors ſhall remaine, BC+BD.

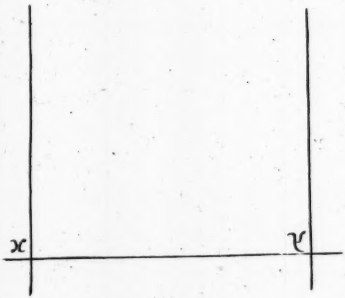
BC exced:	BD defect:
A-D	A+C
BCA-BCD	BDA+BCD.

Here alſo thoſe which are equall being expunged by Addition: the Reduction comes to be BCA+BDA: which is the very ſumme of the Errors drawne into A.

Wherefore $\frac{BCA+BDA}{BC+BD} = A.$

FINIS.


x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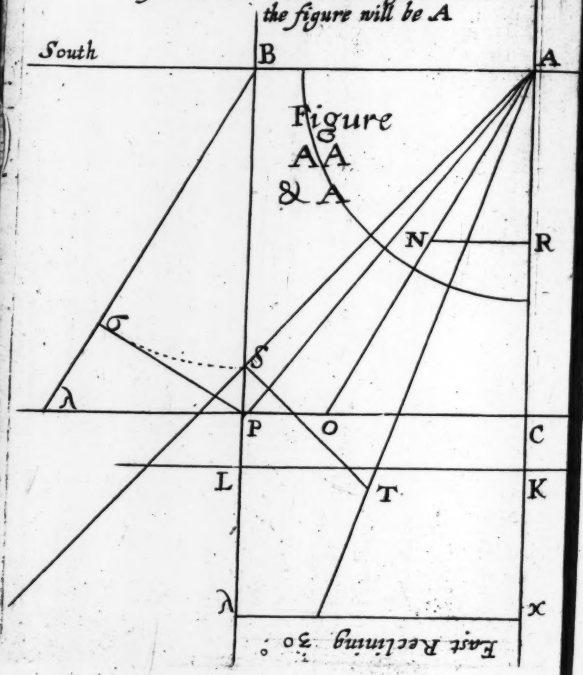
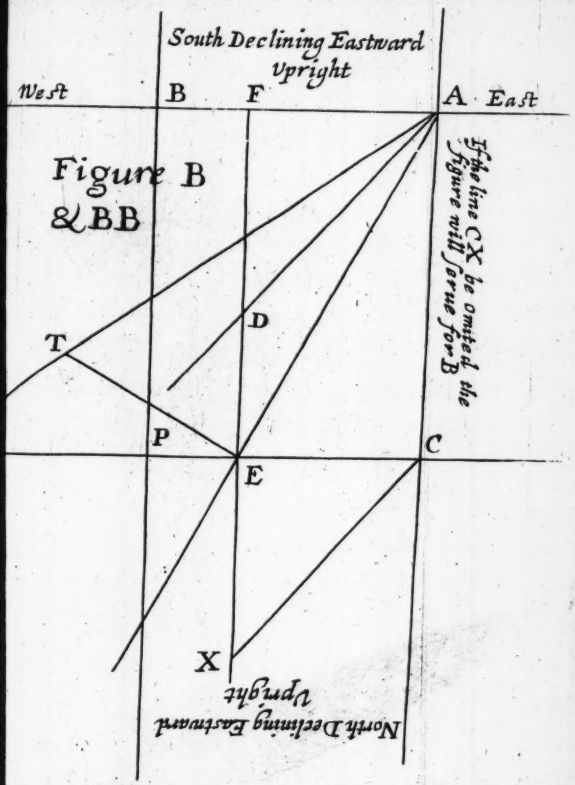
East Direct Inclining degr: 30

If the line x λ and the triangle B P λ be omitted: the figure will be A




South Declining Eastward
Upright
West
B
F
A
East
Figure B
&BB
If the line CX be omited the
figure will serue for B
T
D
P
C
E
X
North Declining Eastward
Upright

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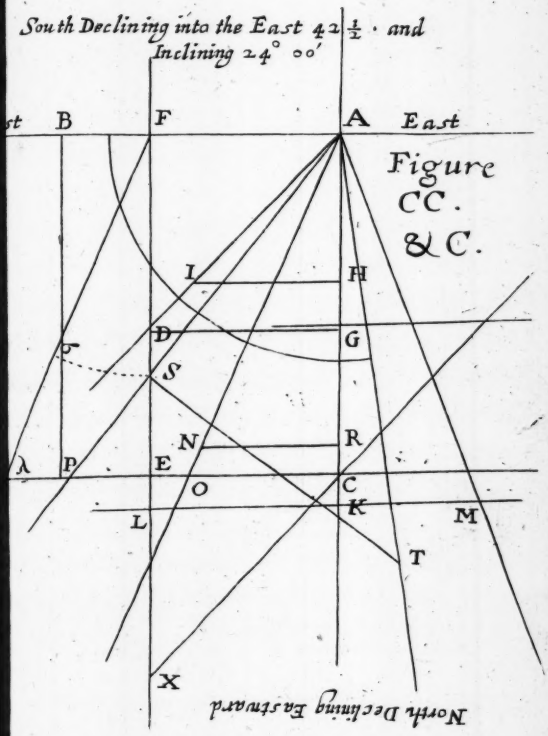
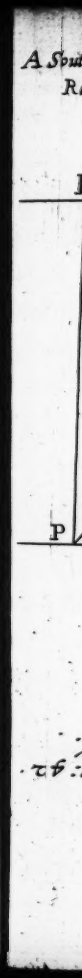
South Declining into the East 42 ½ · and Inclining 24° 00′




A Sou
R
P

A South Dial Declining Eastward Gr: 42 $\frac{1}{2}$.
Reclining Gr: 18 Latitud 51° 30'.

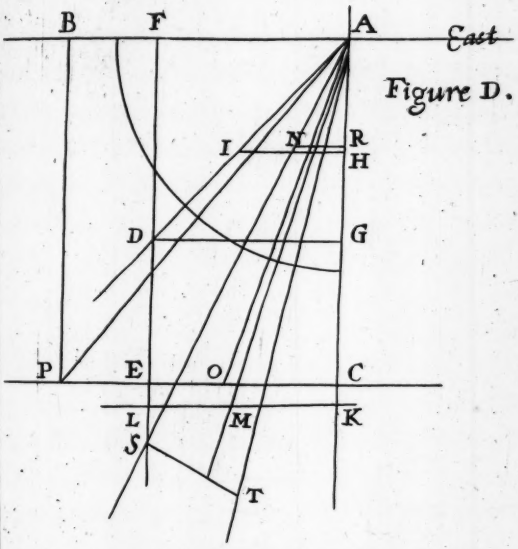
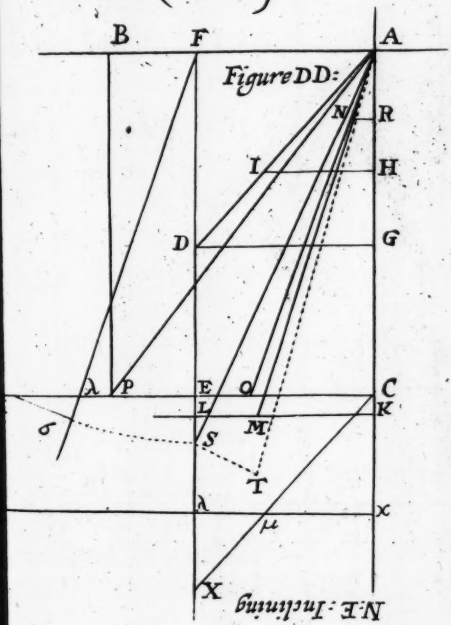
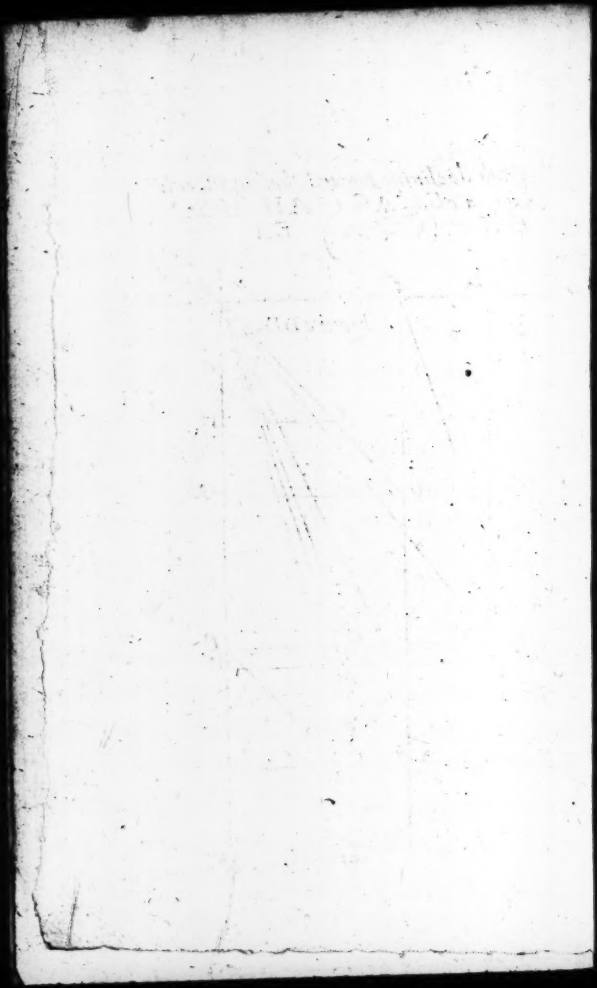


Figure D.

A North Dial Declining Eastward Gr: 42.
Inclining Gr: 18. Latitud 51° 30'.

South declining toward the East Reclining, in which $AG \sqsubset AH$ *that is* $EX \sqsubset (CO =) E\lambda$




AS
Rec
I
O
N
Bun

A South Dial Declining Eastward Gr: 42 ½
Reclining Gr: 40 ½ Latitud 51° 30′.

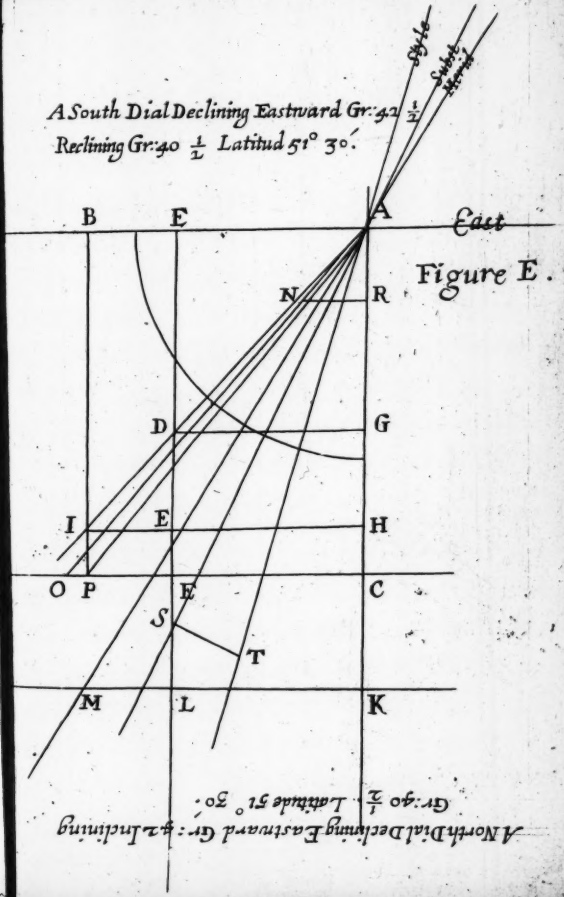


Figure E.

A North Dial Declining Eastward Gr: 42 ½ Inclining
Gr: 40 ½ Latitude 51° 30′.

B
A

South declining toward the East Reclining in which AG ⊏ AH that is EX ⊏ (CO =) Eλ

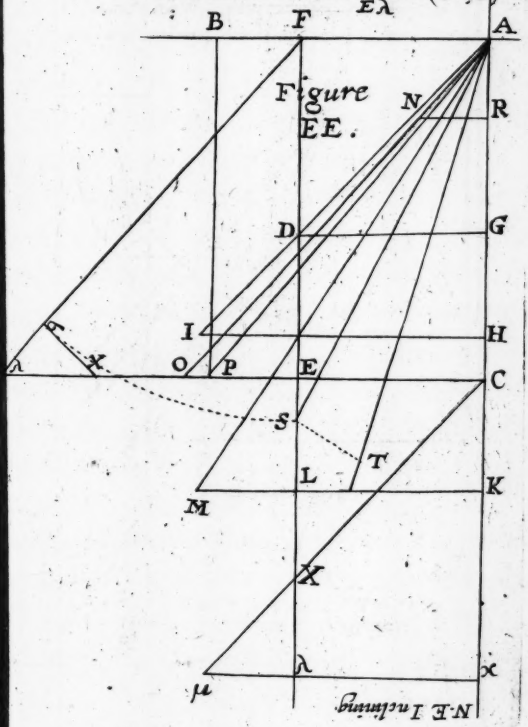


Figure EE.

B
P

A South Diall declining Eastward Gr.: 42 ½.
Reclining Gr.: 30 + Latitud 51° 30'.

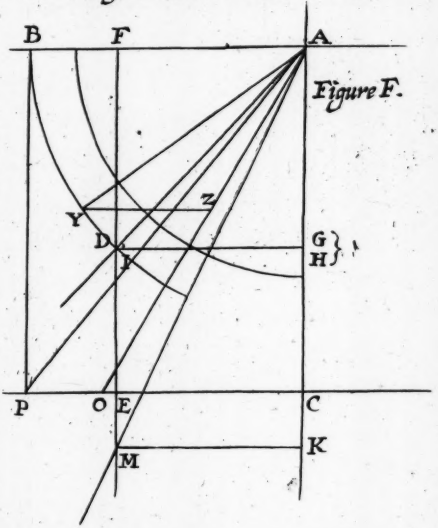


Figure F.

A North Diall declining Eastward Gr.: 42 ½.
Inclining Gr.: 30 + Latitud 51 30.

South declining toward the East Reclining in the which G & H fall in the same point, that is X & λ or in which CO = EX

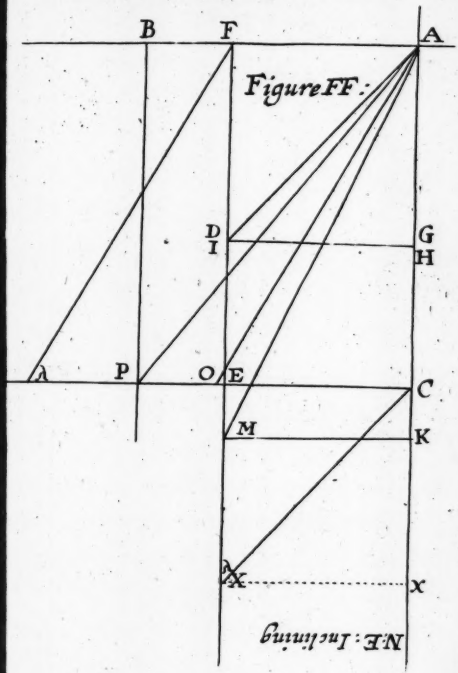
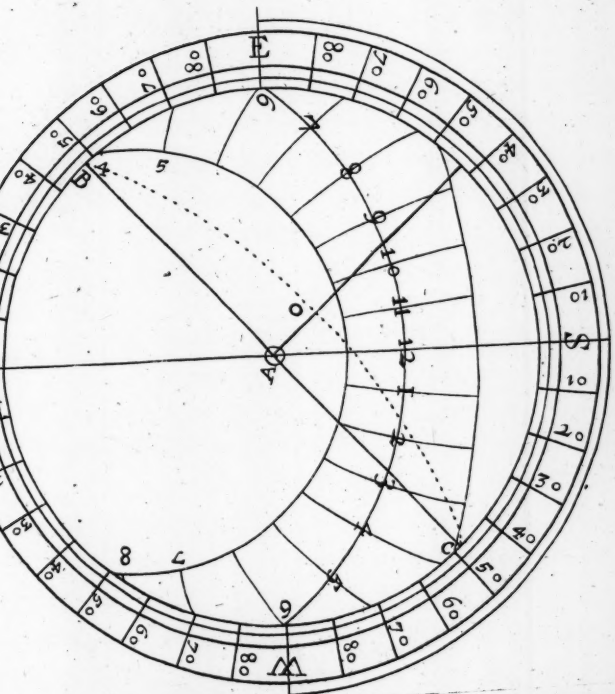


Figure FF:

60
50
40
30
20
10
N
10
20
30
40
50
60



on which the Diall is delineated, is supposed to be parallel to some one great circle of

 the

C
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20
30
40
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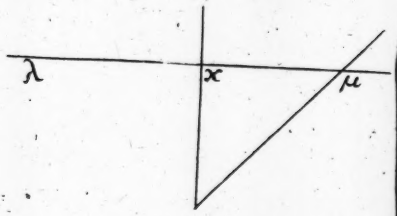
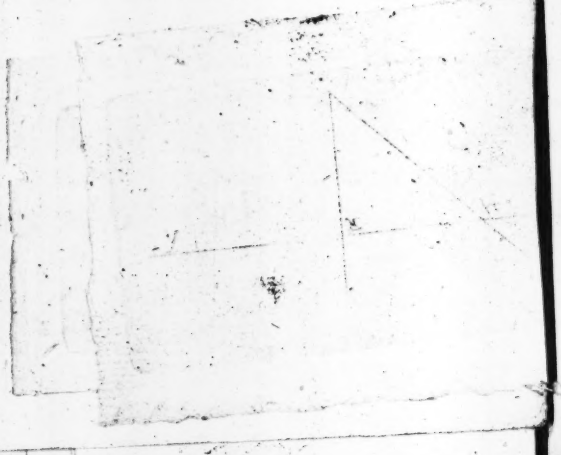
60
70
80
90

W
A
E
T
M


λ
x
μ



A moſt eaſie way for the delineation of Plaine Sun-dials, only by Geometry, without any Trigonometricall calculation, whereby the Meridian, Subſtilar and Style, are not only found out, but alſo inſcribed in every kinde of Plaine in their juſt places.

All plainly demonſtrated; Invented by the AUTHOR, between 22 and 23. yeares of his age.

CHAP. I.

Concerning the Plaines.

THE maine ground of Dialling is the Aſtronomicall ſuppoſition, that the Earth is of no ſenſible quantity, but only as a prick in reſpect of the ſpheer of the Sunne. For the Plain on which the Diall is delineated, is ſuppoſed to be parallel to ſome one great circle of

the heavens, distant from it so farre, as any point which you please to assigne, for the top of the Style is from the said Plaine.

In the Plaine the first thing to be considered, is the situation of it: which is either in respect of the Horizon, or of the Meridian.

In respect of the Horizon, the Plaine is either Parallel, and the Diall made thereon, is called an Horizontall Diall. Or Perpendicular, of which sort all upright walls are. Or Oblique: and so either bendeth forward, and is called Inclining: or else backward, and is called Reclining.

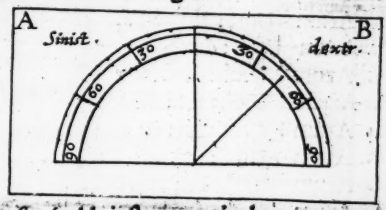
This Obliquity of Inclination or Reclination, is the arch of an Azumith or verticall circle, intercepted between the Zenith of the place, and the Plaine: which Azumith is perpendicular to the Plaine. And is most easily found out by a quadrant divided into 90 parts or degrees.

In respect of the Meridian, the Plaine is either Direct or Declining. The Direct Plaine is that which standeth directly towards one of the foure cardinall points; And is either perpendicular to the Meridian, as are the South and North Plaines: Or Parallel, as are the East and West Plaines. The declining Plaine is that which standeth not directly opposite to any of those cardinall points: but declineth or turneth from the South or North, either Eastward or Westward.

The Declination of a Plaine is the arch of the Horizon

rizon intercepted, betweene the Horizontall Section of the plaine, and the East or West points. Or it is the arch of the Horizon intercepted between the Meridian, and the Pole of that Horizontall Section.

The finding out of the Declination of any Plaine or Wall, is somewhat more difficult. The safest way (because the magneticall needle is subject to bee drawne awry) I take to be by a rectangular boord, about 12 inches long, and 6 broad, with halfe a circle on it, divided from the middle both wayes into 90 degrees: and a strait wyer standing upright in the center, as is seen in this figure.



The use of this instrument is thus: upon any day (the Sunnes declination being first knowne) before Ten a clocke AM, that is before noone; or after Two a clocke PM, that is afternoone; apply unto the wall, the side of the instrument AB, holding it parallel to the Horizon: marke what degree the shadow of the wyer cutteth; either in the right, or the left quadrant, which accordingly I call *umb: dex:* or *umb: sin.* Then instantly take the height of the Sunne. So having the distance of the Sunne both from the North Pole, and

from the Zenith, together with the complement of the Poles altitude, ſeeke the Sunnes Azumith from the South (by the *Analemma* or by the Horizontall inſtrument, or elſe by Trigonometry.) Laſtly with the time of the day, the Sunnes Azumith, and the ſhadow of the wyer, enter the little Table following, and doe as is therein directed: and ſo ſhall you have the true ſituation of the wall.

AM. Azum−Umb : ſin
AM. Azum+Umb:dex } SE.
PM. Umb:dex−Azum
AM. Umb : ſin −Azum
PM. Azum + Umb: ſin } SW.
PM. Azum−Umb : dex
AM. Azum + Umb: dex: ex 180
PM. Azum + Umb: ſin : mi 180 } NE.
PM. Azum + Umb: ſin : ex 180
AM. Azum + Umb: dex:mi 180 } NW.
AM. Azum + Umb: ſin: =90
AM. Azum − Umb: dex=90 } E.
PM. Azum − Umb: ſin =90
PM. Azum + Umb: dex=90 } W.
Azum − Umbra=0 . S.
Azum + Umbra=180. N.

An Example, upon July 15 in the afternoone, I found the ſhadow to be 30 deg: in the right quadrant and Altit: ☉ 23.degr: the Sun being in Northerne declination 20 deg: almoſt. So that the Azumith was

$91\frac{1}{2}$deg. But by the table, PM. Azum-Umb:dex: is SW. Wherefore $91\frac{1}{2}-30$, that is $61\frac{1}{2}$ deg: is the declination SW of the wall.

Another Example, upon July 15 in the afternoon, I found the ſhadow to be 57 deg: in the left quadrant and Altit: ⊙ $22\frac{1}{10}$ which gave the Azumith to be 93 deg. But PM. Azum: + Umb: ſin: ex 180, is NW. Wherefore 93+57 ex 180, that is 30 deg: is the declination NW of the wall.

According to the ſituation of the plaine or wall in reſpect both of the Horizon, and of the Meridian, the Diall made thereon is named. As if the plaine ſtand Reclining, and Declining from the South toward the Eaſt; the Diall thereon is called A South reclining Diall, declining Eaſtward.

Cap. II.

The declaration of the principall lines uſed in the deſcription of Sunne Dials.

1. THE Houre lines are the interſections of the plaines of the Horary circles, with the plaine of the Diall.

2. The principall Howre line is the Meridian or Twelve a clocke line: which is the interſection of the plaine of the Meridian of the place wherein you are, with the plaine of the Diall. And at this the diviſion of the houre lines doth begin.

3. The plaines of the horary circles are all perpendicular to the plaine of the Æquinoctiall; dividing it in 24 equall parts, by ſtreight lines, which are the howre lines of the Æquinoctiall. But they divide all other plaines unequally. And the common interſection of theſe horary plaines, is in the Poles and Axis of the world, or Æquinoctiall.

4. The Style of the Dial (I meane that line which giveth the ſhadow) is underſtood to be a ſegment of the Axis of the world. And therefore ſtill ſo to bee placed that the ends thereof may directly point at the Poles of the world: namely the upper end to the apparent pole; and the lower to the occult.

5. Wherefore, If the plaine cut the Axis of the world, the Dial made thereon ſhall have a center, out of which all the houre lines are drawne. But if the plaine lye parallel to the Axis, the Dial ſhall have no center: but all the houre lines ſhall be parallel both to the Style, and alſo to one another.

6. The Subſtylar is the neareſt line in the plaine, to the Style, and over which the Style hangeth perpendicularly. It is alſo the Meridian of that place of the earth, unto which the plaine is Horizontall: diſtant Eaſtward from you, if the Subſtylar fall among the forenoone houres; or Weſtward, if among the afternoone. The difference of Longitudes is the arch of the Æquinoctiall intercepted between the Subſtylar and the Meridian of the Æquinoctiall.

7. The height of the Pole above the plaine of the Dial,

Dial, is the angle that the Style maketh with the Subſtylar.

8. There is alſo another line of ſpeciall uſe, which is the interſection of the plaine of the Æquinoctiall with the plaine of the Dial, vulgarly called the *Contingent* line; For in it only the howre lines of the Dial interſect the houre lines of the Æquinoctiall. And becauſe the center of Æquinoctial is in the Axis, this line cutteth the Subſtylar at Right-angles.

9. In all Southern plains, the South Pole is elevated: And in all Northerne plaines, the North Pole: Except only in two Caſes, as in their place ſhall be ſhewed: in which the contrary Pole is raiſed: And therefore in them the Subſtylar and Style, being found, muſt be drawne through the center to the oppoſit part.

10. The Delineation of a Dial is performed by three ſeverall operations, in this order. The firſt is, to inſcribe in the plaine, the Meridian, the Subſtylar, and the Style, in the due places. The ſecond is, to draw the *Contingent* line, and the Æquinoctial, with the Meridian, and other houre lines thereof; to bee continued as farre as the Contingent. The third is, to deſcribe the houre lines of the Dial, and to note them with the proper figures.

11. The ſame inſcription of the Meridian, Subſtylar, and Style, ſerveth for the delineation ſometimes of two kindes of Dials, either by turning the paper on which they are drawne, upſide downe, as in all North and South Direct Dials: Or by turning the

backside thereof, as in East and West upright Dials: And sometimes of fowre kindes of Dials, by turning the paper upside downe on both Sides, as in all the rest.

Chap III.

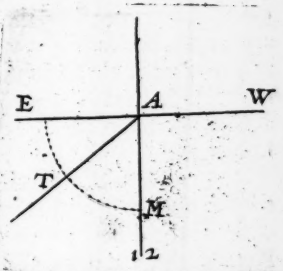
Of the Horizontall Dial.

1. IN an Horizontall Plaine, the Meridian or XII a clock line, is a line drawn exactly from North to South: and therefore lying under the Meridian of the place. It is also the Substylar. And the angle of the Style above it, is equall to the height of the Pole, or latitude of the place.

2. Wherefore in delineating it, draw on the plaine a line directly East and West. Crosse this in the middle about the Point A, with a perpendicular line AM; which shall be both the Meridian and also Substylar. And that point A shall be the Center of the Diall, And the first line E W the VI a clocke line.

Setting

Setting one foot of your Compaſſes in the point A, with the other foot on which ſide you will, deſcribe a quadrant or quarter of a Circle: and on it, from the Meridian, reckon MT an arch equall to the height of the Pole, and at the end thereof out of the Center A, produce a line AT, which ſhall be for the ſtyle.



CHAP. IV.

Of all direct North and South Dialls, whether they be upright or oblique.

1. IN all direct North and South Plaines, both upright and oblique, the Meridian is perpendicular to a line parallel to the Horizon: and it is alſo the Subſtylar.

2. If the Plaine be upright, the height of the Style above the Subſtylar, is equall to the Complement of the Poles height.

3. If the plaine be South Inclining, or North Reclining, the height of the Style above the Subſtylar, is equall to the aggregate of the Complement of the Poles height, and of the Obliquity. But if the Obliquity

quity bee greater then the height of the Pole, the angle of the Styles height, will be greater then a right angle. And if the Obliquity be equall to the height of the Pole, the Plaine lyeth parallel to the Æquinoctiall: wherefore the Style ſhall ſtand upright in the Center A.

4. If the Plaine be South Reclining, or North Inclining, the height of the Style above the Subſtylar, is equall to the difference of the Complement of the Poles height, and of the Obliquity. But if the Obliquity bee greater then the Complement of the Pole, the contrary Pole is elevated, which is one of the two Caſes mentioned in Chap. 2. Sect. 9. And if the Obliquity be equall to the Complement of the Pole, the Plaine lyeth parallel to the Axis: ſo that the Diall deſcribed thereon, ſhall have no Center: as was ſhewed in Chap: 2. Sect: 5.

5. Wherefore in delineating any of theſe kinde of Dialls, draw on the Plaine a line parallel to the Horizon, which is alſo directly Eaſt or Weſt, croſſe this about the middle in the Point A, with a perpendicular line AM, which ſhall be both the Meridian and the Subſtylar. And that point A ſhall be the Center of the Diall, if it have a Center: and the firſt line EW the VI a clocke line, if any be. Setting one foot of your Compaſſes in the Center A; with the other foot, on which ſide you will, deſcribe a Quadrant, under the firſt line EW, if the Plaine be South; or above it, if North. And on that Quadrant from the Meridian,

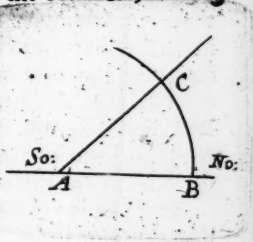
dian, reckon MT, an arch equall to the height of the Style, above the Subſtylar, found out by Sect: 2, 3, 4. And at the end thereof out of the Center A produce a line AT, which ſhall be for the Style, as in the former Scheame, only the letters E and W being tranſpoſed.

CHAP. V.

Of direct Eaſt and Weſt upright Dials.

1. IN direct Eaſt and Weſt upright Dials, there is neither Center, nor Meridian; becauſe that Plain is parallel to the Plaine of the Meridian. But the Subſtylar inſiſteth upon a line parallel to the Horizon, pointing up towards the North, with an angle equall to the Altitude of the Pole, and the Style hangeth parallel over it.

Wherefore in delineating one of theſe Dials, draw on the Plaine a line parallel to the Horizon, noting therein the North and South ends. Then taking in it a Center A, neare to the South end, deſcribe a Quarter of a Circle towards the North: And having taken therein an Arch BC, equall to the Altitude of the Pole produce a line AC for the Subſtylar.



CHAP.

CHAP. VI.

To inscribe the Meridian, Substylar and Style, in Plaines directly East and West, Inclining or Reclining.

1. FIrst draw on the Plaine of the Meridian, a line parallel to the Horizon, noting therein the North and South ends. Crosse this about the middle with a Perpendicular line AC. The Point A shall be the Center of the Diall. Setting one foot of your Compasses in the Center A, with the other foot, unto the line AC of the Diameter, describe a Quadrant, under the first line on the South side, if the plaine be Inclining; or about the first line on the North side, if the plaine be Reclining: And beginning at the Diameter AC, in the proper Quadrant reckon both the Obliquity, and also the Complement of the Poles Altitude. And through the ends of those two Arches, out of the Center A, produce two lines; calling one the line of Obliquity, the other the Polar line. Then in the first line towards the proper Quadrant, take a convenient segment AB; and through the point B draw a line parallel to the Diameter, intercepting the Polar line in P; and so with a fourth line PC parallel to AB, and cutting the line of Obliquity in the point O, conclude the Parallelogram ABPC.

Next measure AK = AO = BL towards CP, and draw the Horizontall line KL.

Lastly,

Laſtly, on the line of Obliquity AO, meaſure AN=CO, and draw NR parallel to AB. Then on the LB toward B, ſet LS=NR. Produce AS for the Subſtylar, upon which in the point S, erect at right angles ST=AR: and produce the line AT for the Style; making with the Subſtylar, the Angle SAT.

2. The demonſtration. Augment the lines AC and BP into ϰ and λ, by adding to them the length of CO, Then make the rect: tri: BPλ=ACO: And cutting the lines thereof Bλ and Pλ, creaſt the lines CP and BP into right angles, in the Inclining backwards and in the Reclining forwards, that the point λ of the triangle may fall upon the other λ in the line Pλ: ſo ſhall the Plaines ACBP, and BPλ inſiſt upon the Horizontall plaine PCϰλ at right angles. And in this poſition you may conceive foure Plaines: namely the Horizontall Plaine PCϰλ, the Erect Plaine ACPB, which is the Plaine of the Meridian, the Oblique Plain ABλϰ=ABLK, the Plain of delineation, becauſe Bλ=BL. Then if from the point P you draw a line Pσ perpendicular to Bλ the Hypotenuſa of the rect: tri: APλ, it is apparent, that the imaginary line Aτ, ſhall be the Subſtylar in the Oblique Plaine, and that the line AS in the Plain of delineation anſwereth to it: And that the height of the Style at the point σ, is Pσ=AR=ST. For the rect: tri: Pσλ=ARN: becauſe the Hypot: Pλ=AN: ang: BλP=ANR is the Complement of Obliquity.

CHAP.

Chap. VII.

To inscribe the Meridian, Substylar, and Style, in all North and South upright Plaines, declining Eastward or Westward.

1. First draw on the Plaine a line parallel to the Horizon, noting therein the East and West ends or sides. Crosse this about the middle in the point A with a perpendicular line AC; which shall be the Meridian. And that point A, the Center of the Diall.

Setting one foot of your Compasses in the Center A, with the other foot unto the line AC, as the Diameter, and upon the side thereof, contrary to the declination, describe a Semicircle: in the lower Quadrant whereof if the Plaine be Southerne; on the upper Quadrant if the Plaine be Northerne; And beginning at the Diameter AC, reckon both the declination, and also the Complement of the Poles height or altitude: And through the end of those two Arches, out of the Center A, produce two lines, calling one the line of declination, the other the Polar line. Then in the first line, toward the Semicircle, take a convenient segment AB; and through the Point B, draw a line parallel to the Diameter, intersecting the Polar line in P. And so with a fourth line PC, parallel to AB, conclude the Parallelogram ABPC. Next on the line of declination measure AP=AB; and through D, draw a line FDE parallel also to the Diameter cutting

ting the line AB in F, and the line PC in E.

Laſtly, produce the line AE for the Subſtylar; upon which in the point E, erect at right angles ET = DF: And produce the line AT for the Style: making with the Subſtylar the angle EAT.

An example of a Diall of an upright Southerne Plaine, declining Eaſtward 42°. 30.

2. The demonſtration. If unto the Parallelogram ACEF, you joyne a tri: CEX = AFP, as it were in a Horizontall Plaine, upon which the Parallelogram ACEF is underſtood to ſtand erect: by creaſting the paper in the line CP, it evidently appeareth that the rect: tri: ACX = ACP, is the Horizontall Gnomon or Cocke: and that the line CX is the Meridian of the Horizontall Plaine; and that the Style AX is the Axis of the world: and that the rect: tri: AET = AEX, is the Gnomon or Cocke of the erect Plaine.

3. And note, that in all declining Plaines, even thoſe alſo which are oblique, the beginning ſtill muſt be with ſuch a figure AFDECX delineated, as hath beene taught, according to the delineation of the wall or Plaine given, drawing alſo therein DG parallel to AF: ſo ſhall AG = XE. This to be once premoniſhed may ſuffice.

CHAP. VIII.

To inſcribe the Meridian, Subſtylar and Style, in Southerne Plaines declining and inclining, or in Northerne Plaines declining and reclining.

1. FIrſt delineate the figure AFDCX according to the Delineation given, as was premoniſhed in Chap. 7, Sect. 3. then beginning alſo at the Diameter AC. reckon on the Semicircle the Obliquity of the Plaine; and having drawne the line of Obliquity AO, ſet AK=AO=FL, towards CE; and produce the Horizontall line LK. Next meaſure AH=CO, and having drawne HI parallel to AB, take upon the Horizontall line KM=HI, but on the contrary ſide of the Diameter. And draw AM for the Meridian.

Laſtly, on the line of Obliquity AO, meaſure AN=AG+AH; and draw NR parallel to AB. Then on the line LF towards F, ſet LS=NR. Produce the line AS for the Subſtylar; upon which point S, erect at right angles ST=AR: And produce the line AT for the Style, making with the Subſtylar the angle SAT.

Example of a Diall on a Southerne Plaine Declining Eaſtward 42°,30, and Inclining 34° 00.

CHAP.

ee Figure C.

But in Figure CC of the demonſtration, the Incli-ation is 24°. oó. Elſe the Figure would be too large.

CHAP IX.

o inſcribe the Meridian, Subſtylar, and Style, in Southerne Plaines Declining and Reclining, or in Northerne Plaines Declining and Inclining.

FIrſt, delineate the Figure AFDECX, according to the declination given as was premoniſhed in hap. 7, Sect: 3.

Then beginning alſo at the Diameter AC; eckon on the Semicircle the Obliquity of the plaine: nd having drawne the line of Obliquity AO, ſet K=AO=FL, towards CE : And draw the Hori-ontall line KL.

Next meaſure AH=CO : and having drawne l parallel to AB, take upon the Horizontall line M=HI, and on the ſame ſide of the Diameter. nd draw AM for the Meridian.

Laſtly, on the line of Obliquity AO, meaſure AN GH, the difference betweene AG and AH : and w NR parallel to AB.

Then if AG⊏AH, that is, EX⊏CO, on the line towards X ſet LS=NR. But if AB ⊐AH, that is, ⊐CO, on the line EF, towards F ſet LS=NR. oduce the line AS for the Subſtylar : upon which the point S, erect all Right-angles ST=AR : And oduce the line AT for the Style, making with the bſtylar the Angle SAT. And in this ſecond caſe.

wherein AG⊐AH, the contrary Pole is elevated; which is another of the two caſes mentioned in Chap. 2, Sect: 9. And if AG=AH, that is EX=CO, the Plaine lyeth parallel to the Axis: So that the Diall deſcribed thereon ſhall have no Center, as was ſhewed in Chap. 2. Sect: 5. And in this caſe AM is the Subſtylar, and not the XII a clocke line.

Example I. Of a Southerne plaine declining Eaſtward 42°36; Reclining 18°.06. *See Figure* D.

Example II. Of a Southerne Plaine, declining Eaſtward 42°.36; Reclining 40°.36. *See Figure* E.

Example III. Of a Southerne Plaine declining Eaſtward 42°.36; Reclining 30 + *See Figure* F.

2 The Demonſtration of the work in Chap: VIII and IX. If the Diall be South Declining Inclining, enlarge the Plaine CEX, by paſting a paper to the line CE, behinde the Plaine ACEP: and therein juſt under the line CA and EF meaſure the ſpaces C and Eλ equall to CO; and through the Point κλ draw a long line. But if the Diall be South Declining Reclining: augment the lines AC and FE unto κ and λ toward X, by adding unto them the length of CO and through the points κλ draw a long line: Then make the rect: tri: FEλ=ACO. And cutting the lines thereof Eλ and Pλ; creaſt the lines CE and FE into Right angles, backwards in South Inclining, but forwards in South Reclining, that the Point λ of the Triangle may fall upon the other λ in the line EλX. So ſhall the plaines ACEP and EPL, inſiſt upon the Hori-

Horizontall plaine XEC$_{\kappa\lambda}$ at Right angles. And in this Poſition you may conceive five Plaines; namely, the Horizontall Plaine XEC$_{\kappa\lambda}$: the erect Plaine ACEF: the plaine of the Meridian ACX, which is the Horizontall Gnomon: the Oblique plaine AF$_{\lambda\kappa}$=AFLK, the plaine of Declination; becauſe FL=F$_{\lambda}$. Then if from the point X you draw a line X$_{\sigma}$ perpendicular to F$_{\lambda}$ the Hypothenuſa of the rect: tri: FE$_{\lambda}$; it is apparent that an imaginary line A$_{\sigma}$ ſhall be the Subſtylar of the Oblique plain; and that the line AS in the plaine of declination anſwers to it: and that the height of the Style at the point σ is X$_{\sigma}$=AR =ST: for the rect: tri: X$_{\sigma\lambda}$=ARN; becauſe the Hypothenuſa X$_{\lambda}$=(AG+AH) AN: and ang: F$_{\lambda}$E =ANR, the Complement of Obliquity. It is apparent alſo that in the ſecond Caſe of Chap: 1 X, wherein AG⊐AH, that is EX⊐E$_{\lambda}$, the contrary Pole is elevated.

Laſtly the Horizontall Meridian CX being extended till it meet with the line $_{\lambda}$X in the point μ, the line $_{\kappa}\mu$ ſhall be equall and parallel to KM=HI: for rect: tri: C$_{\kappa}\mu$=AHI; becauſe C$_{\kappa}$=CO=AH, and ang: $_{\kappa}$Cμ=ECX=HAI.

See Figures with double letters CC.DD.EE.FF. *anſwering to the ſingle* C.D.E.F.

CHAP. X.

To draw the Contingent line, and the Æquinoctiall, with the Meridian, and other howre lines thereof.

1. HAving by the former Rules reſpectively, according to the ſituation of your plaine, inſcribed therein the Meridian AM, the Subſtylar AS, and the Style AT; take in the Subſtylar (as in reaſon you ſhall ſee fit) any point Q: and through it at right angles protract a long line, noting the Eaſterly end thereof with E, and the Weſterly with W. This line EQW is vulgarly called the Contingent line: but is indeed the only common interſection of the plain of the Æquinoctiall with the plain of the Diall, where this line croſſeth the Meridian AM, ſet the letter N.

2. The center of the Æquinoctiall Æ, out of which it is to bee deſcribed, in Dialls which have a center, is a point in the Subſtylar, ſo farre diſtant from the point Q, as the diſtance of Q from the neareſt point of the Style ſhall bee found with your compaſſes. But if the Diall have no center, you may take what point of the Subſtylar you pleaſe for the center of the Æquinoctiall: obſerving herein ſtill this Rule, that looke how farre the center of the Æquinoctiall is from the Contingent, ſo farre diſtant muſt the Style hang parallel over the Subſtylar.

3. When

3. When you have thus found the center of the Æquinoctiall Æ, deſcribe hereupon, toward the Contingent line, one halfe circle of the Æquinoctiall, that is, On each ſide of the Subſtylar a quadrant, of what quantity you pleaſe. Then lay your ruler unto the two points Æ and N, and draw the line ÆN, cutting the Æquinoctiall circle in *m*. This line E*m* ſhall be the Meridian of the Æquinoctiall: at which you are to begin the dividing of the Æquinoctiall circle, both wayes into houres, by arches of 15 degrees: or into halfe houres, by halfe ſuch arches. And through every one of thoſe diviſions, out of the center Æ, produce obſcure lines, terminating at the Contingent. Theſe lines ſhall bee the houre lines of the Æquinoctiall.

4. Hence ariſe theſe Conſectaries. Firſt that, In all Dialls, wherein the ſame line is both Meridian and Subſtylar; the ſame line alſo is the Meridian of the Æquinoctiall.

Secondly, That in Eaſt and Weſt upright Dialls, the Meridian of the Æquinoctiall is that diameter thereof, which is parallel to the Contingent line.

Thirdly, That the arch of the Æquinoctiall betweene the Subſtylar and the Meridian thereof, is the difference of Longitude, or of the Meridian of the place where you are, and of that place of the Earth, to which the plaine is Horizontall, which place evermore lyeth upon that hand of the Subſtylar, on which the Meridian is, namely that way unto which the

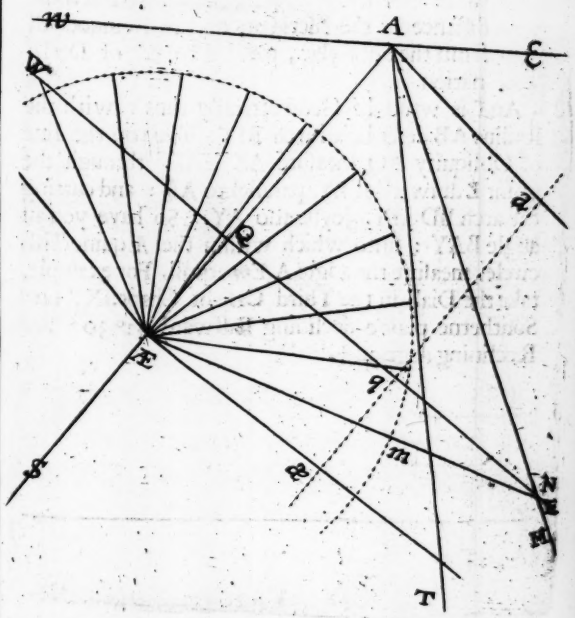
Declination is either East or West. And if this arch be nothing, the Meridian of both places is the same; and they differ only in Latitude.

Fourthly, That the East and West upright Dialls are Horizontall to those, which dwell under the Æquinoctiall, just 90 degrees from your Meridian, either Eastward or Westward.

Lastly, That the East and West Inclining or Reclining Dialls, have a Meridian not 90 degrees distant from yours: and the more Oblique they are, the lesse the Meridians differ.

5. We will take for example the Diall in Chapt. VIII, on a Southerne plaine declining Eastward 42° 36: and Inclining 24° 06: at the figure C. I shall not need punctually to set downe the practice, which is sufficiently delivered before in this Chapter. Only it shall suffice to describe the lines with their notes or markes.

6. But



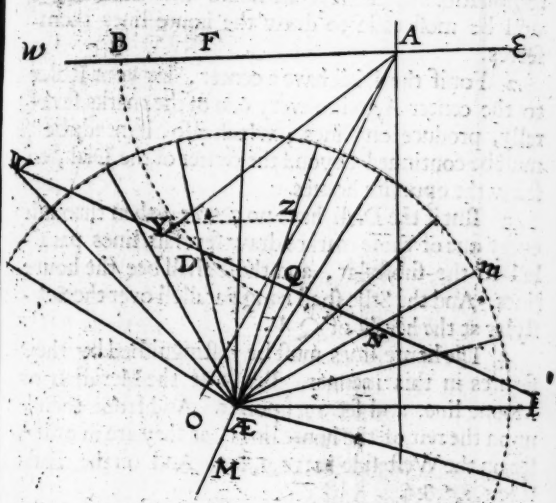
6. But in the Third Caſe of Chapt. IX, becauſe AM is the Subſtylar, and not the XII a clocke line, the Meridian of the Æquinoctiall hath its peculiar way of invention: according to this Theoreme.

As the *Radius* is to the *Sinus* of Declination: So is the *Sinus* of the Complement of Obliquity,

to an arch of the Æquinoctiall : which is the diſtance of the Meridian of the Æquinoctiall from the Subſtylar, toward the ſide of Declination.

And is wrought Geometrically thus : with the Radius AB deſcribe an arch BD. Then on the line of Obliquity AO meaſure AZ=AF. through the point Z draw a line ZY parallel to AB : and cutting the arch BD in Y. joyne alſo AY. So have you an angle BAY : unto which within the Æquinoctiall circle, meaſure the angle AÆ*m* equall. For example, take the Diall in the Third Caſe of Chapt:IX, on a Southerne plaine declining Eaſtward 42° 36 : and Reclining degr:30 †.

CHAP. XI.



CHAP. XI.

To describe the Houre-lines : and to note them with their proper figures.

1. BEcause the Contingent line EQW, is the onely line common to the plaines both of the Æquinoctiall and of the Diall : and you have therein the termi-

terminations of the ſeverall houre lines marked; it will be moſt eaſie to draw the houre lines themſelves.

2 For if the Diall have a center, lay your Ruler to the center A, and to every one of the marks ſeverally, produce long lines; which alſo, if neede bee, muſt be continued beyond the center of the Dial, to ſhew the oppoſite houres.

3 But if the Diall have no center, then through every one of thoſe markes draw ſeverall lines parallell to the ſubſtylar, and thoſe ſhall bee the houre lines. And the Stile ſhall hang parallell over the Subſtylar at the height of Q Æ.

4 The houre lines muſt be diſtinguiſhed by their figures in this manner. Begin at the Meridian or Moone line, and ſet 12 upon it. And from thence upon the reſt of the houre lines, as they are in order, ſet on the Weſt-ſide 11,10,9, &c. And on the Eaſt-ſide 1,2,3, &c.

5 And herein it is commendable, if you deſcribe no more houre lines, then ſhall at ſome times in the yeare ſerve for uſe. Which is found out by an inſtrument of the Horizontall proſection; wherein is inſcribed only the Meridian, Æquinoctiall, two Tropics, and the arches of the horarie circles, betweene the Tropics: And having affixed to the Center a moveable Diameter BAC, with a perpendicular Radius A 90, divided into Degrees. As in the Scheme is to bee ſeene.

The

The uſe of the inſtrument is thus : note on the Radius the degree of Obliquity with ſome blinde marke O. And if the plaine bee Inclining, ſet the Radius to the degree of Declination in the limbe, on the proper ſide : or if the plaine be Reclining, ſet the Radius to the oppoſite degree. Then through both ends of the moveable Diameter, and the point of Obliquity O, imagine an arch of a Circle AOB to bee traced. This arch ſhall repreſent your plaine, Inclining on the convexe ſide; or Reclining on the concave ſide. And ſhall accordingly ſhew what houres are to be deſcribed in your Diall.

For example, Suppoſe a Southerne plaine Declining Eaſtward 42° 30, and Inclining degr: 24. or a Northerne plaine Declining weſtward 42°30, and Reclining degr:24. Set the Radius to 42°30 on the South Eaſt-ſide : and note on it the Obliquity with the letter O. Then through theſe three points given B,O,C, imagine a blinde arch to be traced. This arch on the convexe ſide, that is in the South Inclining Diall, ſhall ſhew from Sunne riſing to one a clocke in the afternoone. And on the concave ſide, that is in the North Reclining Diall, ſhall ſhew from two a clocke in the afternoone till Sun ſetting.

And note that the moveable Diameter repreſenteth an upright wall or plaine having that Declination 42° 30. And after this very manner in all other ſituations of your plaine, both for Declination, and Obliquity, you are to worke with your Inſtrument.

6 If one of the Houre lines either of the Æquinoctiall, or of the Diall given, will not meet with the Contingent line within your paper; ſo that you have not a point of interſection to draw the congruent houre line by: Then croſſe the Contingent, through the point q taken at all adventures, draw a line parallel to the Subſtylar, cutting the houre line given. So have you three lines, namely ÆQ the diſtance of the Center of the Æquinoctial from the Contingent, and AQ the diſtance of the Center of the Diall from the Contingent, and the ſegment of the parallel betweene the Contingent and the houre lines given, to finde out a fourth, which ſhall bee the ſegment of the ſame parallel betweene the Contingent and the Congruent houre line ſought. As in the Scheme in Chapt: X, Sect: 5. ÆQ. AQ:: qæ.qa.

Or AQ. ÆQ:: qa.qæ.

7 Becauſe in the Dialls of Chapt: VIII, the point S may fall to be ſo neat to A, that the Subſtylar cannot certainly bee drawne: you may finde out the angle CAS by the Tables of Triangles, thus,

As the *Sinus* of half the aggregate of the complements of the Poles height and Obliquity, is to the *Sinus* of the difference thereof: ſo is the *Tangent* of halfe the complement of Declination, to the *Tangent* of a firſt arch. Againe,

As the *Sinus* of halfe the Aggregate of the Poles height and the Obliquity, is to the *Sinus* of their difference: ſo is the *Tangent* of halfe the complement

ment of Declination, to the *Tangent* of a ſecond Arch. Then if the Poles height exceed the Obliquity, the Summe of theſe two Arches ſhall be equall to the Angle CAS: Otherwiſe the Difference.

8 Alſo in the Dialls of Chap: VIII. IX. If the CAM (which is for the Meridian) bee hard to bee found: Say,

Rad. s Obliq:: t:Decl. tang:CAM.

Chap. 6. { Page 13. line 7. adde, *See Figure* A.
{ Ibid. Pag. laſt line, adde, *See Figure* AA.
Chap. 7. Pag. 15. line 8. adde, *See Figure* B.

Soli Deo laus & gloria.

FINIS.